

FREGE ON IDENTITY

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What is the identity relation? Is it a relation between names, denotations of names, or senses of names? Frege's answer has been the subject of much dispute, and this dispute is the main topic of the present article. But an article devoted to what identity is, according to Frege, would be incomplete if his view of senses, as well as his explanation of identity as indiscernibility, were not at least briefly discussed as well. I shall therefore also discuss these topics.

It is fairly clear that in *Begriffsschrift* Frege held that identity is a relation between names, i.e., that " $a = b$ " has the same content as does "The names ' a ' and ' b ' stand for the same content."¹ What is in question is whether Frege later changed his mind. Most philosophers seem to agree that Frege's major later statement on the topic begins with a rejection of the *Begriffsschrift* view. But what the rejection amounts to, and what other view Frege is led to, are far from clear. I am referring, of course, to the first two paragraphs of "On Sense and Reference."² I shall state and defend my own interpretation of this passage first, and examine the other interpretations later.

My view is that while identity remains a relation between names, now names are understood differently. In *Begriffsschrift*, names were mere labels. Now they are signs expressing senses. " $a = b$ " is now to be understood as expressing the thought expressed by "The names ' a ' and ' b ', which are understood to be signs expressing senses, denote the same denotation." In *Begriffsschrift* names, instead of expressing senses, correspond to ways of determining the contents they stand for.³ A sense is a kind of unit of meaning in virtue of being part of a thought, where thoughts are what we express in language (a thought is itself a sense).⁴ But a way of determining a content is not put forward by Frege as being the meaning of any name it is put in correspondence with.⁵ A way of determining a content seems best identified in "On Sense and Reference" not with a sense, but with the mode of presentation which is contained within a sense.⁶ But even here there seems to be a difference: a way of determining a content is, as it were, on the side of the observer; it is a procedure one may employ, while a mode of presentation is on the side of what is observed: it is how something can present itself to us.

That is my interpretation. I argue for it as follows. In the first part of the "On Sense and Reference" passage, Frege repeats the argument and position of *Begriffsschrift*. If identity were a relation between the objects named (technically between the object and itself), then an informative identity would be impos-

sible. But informative identities are possible, as is proved by their being so often actual. Therefore identity is not a relation between the objects.⁷ So far so good; what follows is what has been so hard to understand, beginning with the sentence, "But this is arbitrary." What is Frege saying now?

I submit that Frege now argues as follows. It is not enough merely that identity be a relation between names, for a difference between names considered as mere labels does not suffice to account for the possible informativeness of an identity. That is, the mere fact that the letters "a" and "b" are different, that they differ in shape, cannot explain the possible informativeness of " $a = b$ ".⁸ (In the language of the completed argument, the reason " $a = b$ " need not be informative is that "a" and "b" can be easily assigned the same sense.) The trouble is that names considered merely as labels are arbitrarily assignable to any object. There is no reason to prefer assigning a mere label to any object as opposed to assigning it to some other object. So that if names are mere labels, then any assertion made about them will have nothing at all to do with the objects they denote. It would then be impossible for an informative identity to be a statement about such names. Frege therefore rejects the *Begriffsschrift* view. But it is important to realize what sort of rejection this is. The whole argument is based on the construal of names as mere labels; it is this construal that gives trouble. Perhaps it is this construal of names that Frege wishes to change.

This is just what happens. We find in "On Sense and Reference" no hint of a return to the rejected view that identity is a relation between objects, nor do we find any hint of a theory that identity is relation between senses, for example the relation of containing modes of presentation of the same object. What we find is a distinction between two construals of names, and this indicates that Frege finds one construal of names inadequate and substitutes another, the substitution being the sole improvement made on the *Begriffsschrift* theory, aside from replacing ways of determining contents with modes of presentation.

Now if on the earlier construal we could not explain how an informative identity is possible, how can we do so on the new construal? Specifically, how is the problem of arbitrariness avoided? A sign that expresses a sense designates in a certain manner, i.e., it relates *qua* linguistic expression to a definite way an object can be presented before consciousness. (With regard to names in a natural language, the ostensibly named objects need not even exist.) Therefore names can no longer be arbitrarily assigned to any object. In fact, a name can now belong in principle to one object at best, even if we might not be sure in practice which object it is, or even whether it exists. As for the undeniable element of arbitrariness in language, this now appears at the level of assigning senses to names. Only in this way, Frege thinks, can pertinence to the subject matter be built into names, and it is this pertinence that ensures the possibility of an informative identity. Without it, we find ourselves once again viewing names as mere labels, and the assertion that two labels in our language denote the same object can hardly be the vehicle by which we learn anything factual about the world.

The transformation of ways of determining the subject matter into modes of presentation is doubtless meant to enhance the pertinence we wish to build into names. A mere procedure on the side of the observer might "determine," for all we know, two objects if we follow it, or none clearly. But a mode of presentation, being on the side of the object, belongs to one object only and belongs to it clearly, if it belongs to an object at all, thanks respectively to the uniqueness⁹ and the determinacy¹⁰ of every object. There is still much room, of course, for difficulties in practice.

I shall proceed to the other interpretations. The one closest to my own is that of Ignacio Angelelli, who draws the same distinction between construals of names that I do. Our only difference concerns Frege's rejection of the view that identity is a relation between objects. Angelelli takes the uninformative nature of an identity on that theory to be the same as the contentlessness of predicates such as "is one" and "exists" when they are viewed as denoting first-level concepts.¹¹ This is not so: "is one," so viewed, is held by Frege to be contentless because all objects would fall under the concept it denotes,¹² but "is identical with" is not held to be contentless because it denotes a relation under which all pairs of objects fall. Far from contentless, identity on the rejected theory is uninformative precisely because it would be a relation a thing had only to itself, so that most (but not all) pairs of objects would be mapped onto the False.

Michael Dummett,¹³ Robert Ray,¹⁴ and P. D. Wienpahl¹⁵ all hold that in "On Sense and Reference," identity is a relation between objects. Dummett gives no argument. Ray sees the passage in the article after the sentence "But this is arbitrary" as giving an argument that identity is not a relation between names, and infers a return to the view that identity is a relation between objects. Ray thinks that Frege's argument is that " $a=a$ " would express an identity fully as informative as the one expressed by " $a=b$ " if identity were a relation between names, since each of the letters, " a " and " a ," flanking the identity sign in " $a=a$," is an arbitrarily producible event and need not stand for the same object the "other" letter stands for.¹⁶ But Frege gives no such argument. It is safe to assume that Frege thinks at least that the same letter always stands for the same object, at least in the same propositional context. This is so even though Frege accepts only tokens, not types.¹⁷ Ray's citing the common fact that in English, "bank" can express different senses on different occasions is especially misguided, since even as early as *Begriffsschrift*, one of Frege's motives in having a formal notation at all was to avoid precisely such ambiguities of natural language.¹⁸ If anything, Ray has got the argument backwards. Frege is arguing not that " $a=a$ " would be as informative as " $a=b$," but that " $a=b$ " would be as uninformative as " $a=a$," if identity were a relation between mere labels.

Ray argues that if Frege's view were that produced objects are the same sign only if they express the same sense, so that the difference in cognitive value between " $a=a$ " and " $a=b$ " might be restored, this view would be false and odd. It would be false because the two subject-terms in " $a=a$ " need not be the same sign; they can be assigned different senses, so that " $a=a$ " remains as

informative as " $a=b$ ".¹⁹ This criticism would be right if Ray's interpretation were right, but we have seen that it is not.

Ray's argument that it would be odd if Frege's view were that produced objects are the same sign only if they express the same sense is that Frege's later theory of identity is that identity is a relation between objects.²⁰ Ray's argument is a little odd itself, since one rarely hears the argument that a philosopher must have held a certain opinion in an early work, since he held it in a later one: he might, for example, have changed his mind. Worse, Ray never argues for his argument's premise, which is that Frege later held that identity is a relation between objects. I presume that Ray has *The Basic Laws of Arithmetic* in mind. Now Montgomery Furth, while not taking sides on "On Sense and Reference," claims that by the time of *The Basic Laws of Arithmetic*, Frege did change from the *Begriffsschrift* view that identity is a relation between names to the view that it is a relation between objects.²¹ Furth has no argument to offer for this view either. He probably bases his claim on vol. 1, sect. 7, which stipulates that " $\Gamma = \Delta$ " is to denote the True if and only if Γ is identical with Δ .²² But this stipulation is not at all an attempt to state whether identity is a relation between names or between objects. Nor is it an attempt to define identity.²³ It is an attempt to make the identity relation determinate, in accordance with vol. 1, sects. 29, 31. To the contrary, in *The Basic Laws of Arithmetic* identity is, if anything, still a relation between names, at least if we can treat as evidence certain casual formulations aside from which I can find no evidence for or against Furth's claim.²⁴

P. D. Wienpahl's argument that in "On Sense and Reference" identity is a relation between objects is an interesting one. He distinguishes three possibilities: identity is a relation between either modes of presentation, names, or objects. He rejects the first possibility because to him it means saying that " $a=b$ " expresses the thought expressed by "The mode of presentation associated with ' a ' is identical with the mode of presentation associated with ' b '," which of course entails that " $a=b$ " is false whenever " a " and " b " express different senses. Wienpahl does not see that " $a=b$ " might be understood as expressing the thought expressed by "The modes of presentation contained in the senses of ' a ' and ' b ' are modes of presentation of the same object." (I shall not discuss this variant of the theory that identity is a relation between senses further; my rejection of the one as being Frege's will apply equally to the other.) Wienpahl sees Frege as finding problems with the two other possibilities. He then says that what Frege does is to go back to the view that identity is a relation between objects, and concludes that the text must be incomplete because it fails to mention this return. Wienpahl has no argument for this view of his.²⁵

The next interpretation I shall examine is offered by David Coder. Coder holds that in "On Sense and Reference," Frege makes no explicit choice between the view that identity is a relation between names and the view that it is a relation between objects. While in *Begriffsschrift* Frege held the first view, he now sees that both views, each in its own way, make an informative identity

impossible.²⁷ Frege therefore chooses neither theory, but merely insists that for an informative identity to be possible, its subject-terms must not only (customarily) denote objects, but must express senses as well.²⁸ Coder's view is a pleasingly moderate one. Frege does not, indeed, pronounce either theory to be correct in so many words. My reason for retaining my own interpretation is that Frege distinguishes between two construals of names, and this distinction is clearly intended to be a refinement of the *Begriffsschrift* view. Since Frege does not revive the theory he first criticizes, nor pronounce the second theory dead, the presence of the distinction I mentioned can only mean that the second theory survives in an altered form.

Leonard Linsky, if I understand him correctly, holds still another view. Unlike Coder, who says that Frege makes no choice between the two alternatives he discusses, Linsky sees Frege as definitely rejecting both in favor of a third view. This view is that names express senses as well as denoting denotations.²⁹ The trouble with this view is that it does not tell us what identity is a relation between. Now the task of explaining the possibility of an informative identity is not the same as the task of determining what identity is a relation between. But the whole problem of how an informative identity is possible arises precisely because this possibility seems incompatible with our *prima facie* belief as to what identity obtains between, namely, objects. The solution to this problem is not merely to introduce the sense-denotation distinction, but instead, if anything, to use that distinction to arrive at some view of what it is that identity obtains between that is compatible with the possibility of an informative identity. This is what the original problem is, and to see Frege as offering no solution to it is to accuse him of a gross oversight.

Nobody in the literature holds that Frege's theory is that " $a=b$ " expresses the thought expressed by "The senses of ' a ' and ' b ' contain modes of presentation of the same object." Yet this view would share with the view that identity is a relation between objects one advantage over my own view. Senses and many objects can be said to enjoy an existence apart from the existence of languages, while names cannot be said to do this, since what makes an object a name is its function in language. How then on my view could the *Morning Star* and the *Evening Star* have been for Frege the same planet before languages were invented? Twenty million years ago there simply were, I assume, no names to stand in the relation of denoting the same object. It is to no avail to say that today we can say that they were the same planet because today we have names. This theory would also allow us to retain our interpretation of Frege's "On Sense and Reference" arguments as being arguments that identity is not a relation between objects and that it is not a relation between labels; the only difference is about which portion Frege finally opts for.

One response to this new theory would be that ours is the one that is justified by the text, even if it is intrinsically a less plausible theory of identity. But I hope that a better response can be made. The alleged ground of superiority is really irrelevant to what Frege is doing. He is offering an analysis of

sentences of the forms " $a=a$ " and " $a=b$." What relevance to this analysis does the question whether certain objects were identical before languages existed have?

Holding as I do that Frege's theory of identity is that " $a=b$ " expresses the thought expressed by "The names ' a ' and ' b ,' understood as signs expressing senses, denote the same denotation," my presentation of this theory would be incomplete if I did not consider directly the question, what are Fregean senses? The most succinct answer is that Frege does not tell us just what they are, though he does tell us that they are senses of words, that they are abstract, timeless, and that they contain modes of presentation of objects, at least when they are senses expressed by proper names. But several more ambitious answers are available in the literature. It has been held that senses are objects.³⁰ that some senses are objects and the rest are functions,³¹ and that senses are neither objects nor functions.³² The first view is obviously wrong because some senses are complete and some are not,³³ while all objects are complete.³⁴ According to the second view, proper names express senses which are objects and function-names express senses which are functions. This view is very plausible in light of Frege's claim that names denote their customary senses when they occur in what Quine would call referentially opaque propositional contexts,³⁵ though Furth, the only one who holds this view, does not argue for it.³⁶

My own inclination is toward the third view for two reasons. First, a sense contains a mode of presentation, and a mode of presentation is a mode of possible cognition. But a mode of cognition is not as such an object of cognition. An object of cognition would be in language a denotation, and can be either a Fregean object or a Fregean property (concept). Now anything that is an object of cognition simply is not functioning as a mode of presentation. That is, if it is presented as an object, a way an entity is presented is no longer a way an entity is presented, but an object of presentation. A similar argument applies to the sense itself: a sense that is presented to us as an object is no longer a sense but an object. Second, if senses were objects or functions, Frege probably would have said so. He has no qualms about telling us all about the different types of objects and functions anywhere else. His talk of senses has instead all the earmarks of an introduction of a category distinct from those of objects and functions.³⁷

It follows from the arguments just given that Frege is mistaken about his own senses in thinking that a sense can ever be denoted by a name. How then can we talk about senses at all? The answer is that if we use what Frege calls the form usual for asserting something of objects, then senses must be represented by objects much in the manner he says concepts are represented by objects.³⁸ Every sense contains a mode of presentation, and every mode of presentation clearly must be intimately associated with some concept under which the ostensible object ostensibly falls. This concept already has an object to represent it, according to Frege, and we can simply use this very object to

make, in addition to representative assertions about concepts, a second sort of representative assertion about senses. Even more simply we can use a name, taken as denoting itself, to represent the sense it customarily expresses. What might be called the problem of the sense of expression *A* is really no more paradoxical than the celebrated problem of the concept *horse*. Frege says that "the concept *horse*" must denote not a concept but an object, and we say that for the very same reason of linguistic form, "the sense of expression *A*" must denote not a sense but an object. Indeed, it is hard to accept Frege's view while not accepting this view too.³⁹ The mutual representation of objects and senses is out of the question, since senses cannot be denoted at all; objects and concepts, of course, can represent each other, since both objects and concepts can be denoted.⁴⁰

I shall make one final point about senses. Since modes of presentation and therefore also senses are on the side of the object, no matter how obviously true a certain identity proposition may be to a given observer in a given situation, the proposition is still informative if, as might easily be the case, the senses involved are different. This shows that Frege means by "informative" not novel but factual. He is explaining only the possibility that someone at some time might be informed by an identity proposition. We would not wish to commit Frege to the view that everyone is always surprised or informed whenever they hear an identity proposition Frege would call informative.⁴¹

My conclusion is that Frege's senses form a special, irreducible, indefinable category of entities, much as his functions and objects do.⁴² If so, it would seem that there is now little else to say about senses. It might seem that there is now little else to say about the identity relation either, since Frege says that it is indefinable,⁴³ but this, I fear, would be mistaken. Frege says that while identity cannot be defined (*Definition*), it can be explained (*Erläuterung*).⁴⁴ What does he have in mind?

What may be called Frege's threefold distinction between definition, explanation, and explication (*Erläuterung*) is as follows. A definition stipulates the sense and denotation of the term defined.⁴⁵ As a stipulation it is neither true nor false.⁴⁶ Not all names can be defined; to suppose that they could would involve us in a vicious infinite regress of definitions.⁴⁷ Therefore, Frege argues, there must be indefinable names. Their meaning is conveyed by explication, which consists of hints, metaphors, and suggestions.⁴⁸ While definitions are foundationally part of science, explications are a propaedeutic to science.⁴⁹ Frege's definitions are intended to guarantee both sense and denotation for the defined term. Explications are not at all guaranteed to provide these for the scientist's audience.⁵⁰ Scientific communication thus always begins with an element of uncertainty.

What is an explanation? In Frege's words Leibniz's "explanation" of identity (it was a definition for Leibniz) conveys something "fundamentally important" about identity to those who already possess that concept.⁵¹ That

distinguishes explanation from both definition and explication. But what does an explanation explain? While a definition provides a sign with a sense and a denotation, in an explanation the explained term is assumed to have a sense and denotation already. At the same time an explanation seems to leave nothing to chance, as an explication would. I suggest that an explanation may be understood as the assertion that two names, both already assumed to have sense and denotation, denote the same entity. An explanation, if correct, is therefore a true proposition. To call Frege's explanation of identity an axiom, as Frege does, is only to add that it cannot be derived from more fundamental truths.⁵² An explanation explains an entity that is presented to us in one way as being the same as an entity that is presented to us in another way. At bottom, it is simply an informative identity. In the case of a relation such as identity a sufficient condition of an explanation's truth may be taken to be that the relations named always yield for the same argument the same value. This, as we know, is a sufficient condition for the fact that a pair of functions are interdefinable (relations are functions), in those cases where definition is allowed,⁵³ and interdefinable functions can hardly be regarded as different functions.

Frege's explanation that identity is indiscernibility, or intersubstitutability *salva veritate* (both relations relate names), therefore commits him to both the indiscernibility of identicals and the identity of indiscernibles. The former principle is commonly regarded as trivial, but the latter is not. It is therefore surprising to learn that Frege officially defends the second principle in a trivial way. Frege argues that if *a* is not identical with *b*, then there is always at least one property *a* has which *b* does not have, namely, the property of being identical to *a*.⁵⁴ Technically this is a legitimate argument in Frege's notational system, since every well-formed predicate denotes a genuine property. But one wishes that Frege had given a more substantial argument. For example, it can be argued that if things could have all properties in common, then it would be impossible to identify or name anything. If there are 100 oranges in a crate, all having all properties in common, even spatiotemporal ones, how could I tell one apart from the rest? How could I tell that there are not 99 but 100? This more serious sort of argument is in fact used by Frege elsewhere. He argues that since we cannot specify any property *x* has that is different from any property of *y*, we cannot conceive of variable numbers each in its individual being, so that we cannot assign proper names to variable numbers (or say that they exist).⁵⁵ I attribute to Frege, therefore, the very serious sort of argument for the identity of indiscernibles that was described in the case of the oranges as being at least tacitly present in his thinking about variable numbers. Indeed it is almost pronounced enough to be called explicit.

¹ Gottlob Frege, *Begriffsschrift, a formula language, modeled upon that of arithmetic, for pure thought*, in Jean van Heijenoort, ed., *From Frege to Gödel*, Cambridge, Massachusetts, Harvard University Press, 1967, pp. 20-21.

2 Frege, "On Sense and Reference," in Peter Geach and Max Black, trans. and eds., *Translations from the Philosophical Writings of Gottlob Frege*, Oxford, Basil Blackwell, 1970, pp. 56-57.

3 *Begriffsschrift*, p. 21.

4 "On Sense and Reference," p. 62.

5 *Begriffsschrift*, p. 21.

6 "On Sense and Reference," p. 57.

7 *Ibid.*, pp. 56-57.

8 *Ibid.*, p. 57.

9 Frege accepts the identity of indiscernibles. Illustrative extracts from Frege's review of Husserl's *Philosophie der Arithmetik*, in *Translations from the Philosophical Writings of Gottlob Frege*, pp. 80-81.

10 Frege, "Function and Concept," in *Translations from the Philosophical Writings of Gottlob Frege*, p. 33.

11 Ignacio Angelelli, *Gottlob Frege and Traditional Philosophy*, Dordrecht, Holland, D. Reidel Publishing Company, 1967, pp. 38-40, 43-44.

12 Frege, *The Foundations of Arithmetic*, J. L. Austin, trans., 2nd rev. ed., Evanston, Northwestern University Press, 1974, p. 40.

13 Michael Dummett, *Frege: Philosophy of Language*, New York, Harper and Row, 1973, p. 544.

14 Robert Ray, "Frege's Difficulties with Identity," *Philosophical Studies*, 31, 1977, p. 232.

15 P. D. Wienpahl, "Frege's *Sinn und Bedeutung*," *Mind*, LIX, 1950, p. 484.

16 "Frege's Difficulties with Identity," p. 230.

17 Frege, *Grundgesetze der Arithmetik*, Vol. ii, sect. 99, in *Translations from the Philosophical Writings of Gottlob Frege*, p. 194.

18 *Begriffsschrift*, p. 11.

19 "Frege's Difficulties with Identity," pp. 231-232.

20 *Ibid.*, p. 232.

21 Montgomery Furth, "Editor's Introduction to Frege," *The Basic Laws of Arithmetic*, Furth trans. and ed., Berkeley, University of California Press, 1967, pp. xviii-xix.

22 *The Basic Laws of Arithmetic*, p. 46.

23 Identity is indefinable, p. 87.

24 *Grundgesetze der Arithmetik*, Vol. i, sect. 5, p. 156 ("Again if I . . ."), and Vol. ii, sect. 64, p. 167 ("If our sentence . . .").

25 "Frege's *Sinn und Bedeutung*," p. 485.

26 *Ibid.*, pp. 484-485.

27 David Coder, "The Opening Passage of Frege's 'Über Sinn und Bedeutung,'" *Philosophia*, Vol. 4/2-3, April-July 1974, pp. 340-341.

28 *Ibid.*, p. 342.

29 Leonard Linsky, *Referring*, London, Routledge and Kegan Paul, 1967, pp. 22-23. Coder, in "The Opening Passage of Frege's 'Über Sinn und Bedeutung,'" p. 343, footnote 1, says that Linsky holds that identity is a relation between objects. Coder cites *Referring*, p. 22. I find no evidence there for his claim.

30 Rulon S. Wells, "Frege's Ontology," *The Review of Metaphysics*, IV, 1951, p. 542; Gustav Bergmann, "Frege's Hidden Nominalism," *The Philosophical Review*, LXVII, 1958, p. 447; Howard Jackson, "Frege's Ontology," *The Philosophical Review*, LXIX, 1960, p. 394.

31 Montgomery Furth, "Two Types of Denotation," in Nicholas Rescher, ed., *Studies in Logical Theory, American Philosophical Quarterly Monograph Series*, Monograph No. 2, Oxford, Basil Blackwell, 1968, p. 9.

32 Eike-Henner W. Kluge, Editor's Introduction, in Kluge, ed., *Frege: On the Foundations of Geometry and Formal Theories of Arithmetic*, New Haven, Yale University Press, 1971, pp. xxxiii-xxxiv. See also Dummett, "Frege on Functions: A Reply," *The Philosophical Review*, LXIV, 1955, p. 98, and p. 98, footnote 5.

33 Frege, "On Concept and Object," in *Translations from the Philosophical Writings of Gottlob Frege*, pp. 54-55.

34 Frege, "On the Foundations of Geometry" (1903), in *Frege: On the Foundations of Geometry and Formal Theories of Arithmetic*, p. 33.

35 Willard Van Orman Quine, "Reference and Modality," in Quine, *From a Logical Point of View*, 2nd ed. rev., Cambridge, Massachusetts, Harvard University Press, 1961, p. 142. See also p. 140, footnote 1.

36 "Two Types of Denotation," p. 9.

37 "On Sense and Reference," pp. 56-61. See also p. 64 ("A truth value . . .").

38 "On Concept and Object," pp. 45-46.

39 The special nature of Fregean representative assertion allows us to see how Frege can use senses to explain the possibility of informative identity propositions without appealing circularly to a special class of such propositions about senses. This would be my way of substantiating Panayot Butchvarov's observation that Frege would not be guilty of such an easy fallacy. See Butchvarov, *Being Qua Being*, Bloomington, Indiana University Press, 1979, p. 25.

40 See *The Basic Laws of Arithmetic*, pp. 92-94, where the object that represents a function is its course-of-values. Concepts, of course, are functions.

41 That Frege holds such a view unhappily seems to be the basis of a major criticism of Frege's theory of identity. See *Being Qua Being*, pp. 21-24.

42 "On Concept and Object," pp. 42-43.

43 Illustrative extracts from Frege's review of Husserl's *Philosophie der Arithmetik*, p. 80.

44 Frege, "Rezension von: E. G. Husserl, *Philosophie der Arithmetik* k. I.," in Ignacio Angelelli, ed., *Gottlob Frege: Kleine Schriften*, Hildesheim, Georg Olm Verlagsbuchhandlung, 1967, p. 184. The explanation Frege gives of identity as indiscernibility was accepted by Frege as a definition earlier in *The Foundations of Arithmetic*, J. L. Austin, trans., Evanston, Northwestern University Press, 1974, p. 76. What made Frege change his mind was apparently only the technical ground that for him, all definitions are identities, so that identity itself must be indefinable.

45 *The Basic Laws of Arithmetic*, p. 82.

46 "On the Foundations of Geometry" (1903), p. 24; letter from Frege to Husserl, in *Frege: On the Foundations of Geometry and Formal Theories of Arithmetic*, p. 7.

47 "On Concept and Object," pp. 42-43; *The Basic Laws of Arithmetic*, p. 85.

48 "On the Foundations of Geometry" (1906), in *Frege: On the Foundations of Geometry and Formal Theories of Arithmetic*, pp. 59-60.

49 *Ibid.*, pp. 59-61; letter from Frege to Husserl, p. 8.

50 "On the Foundations of Geometry" (1906), p. 59; letter from Frege to Husserl, pp. 8-9.

51 Illustrative extracts from Frege's review of Husserl's *Philosophie der Arithmetik*, p. 81.

52 "On the Foundations of Geometry" (1903), pp. 23-25. The fact that the explanation under discussion is an axiom should provide at least part of the rationale behind Frege's saying that it is of fundamental importance. Frege might possibly also mean in part that identity is in some sense essentially indiscernibility, or that the axiom is fundamental philosophically speaking.

53 Illustrative extracts from Frege's review of Husserl's *Philosophie der Arithmetik*, p. 80. The relations of identity and of intersubstitutability *salva veritate* always yield for the same argument the same value, but Frege has a special reason for holding that identity is indefinable.

54 *The Basic Laws of Arithmetic*, p. 71.

55 Frege, "What is a Function?," in *Translations from the Philosophical Writings of Gottlob Frege*, p. 109.