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DUMMETT'S BACKWARD ROAD TO FREGE AND TO INTUITIONISM

This essay addresses Michael Dummett's paper, "The Context Principle: Centre of Frege's Philosophy" (1995), in which Dummett revises his thinking on Frege. I shall argue that Dummett's semantic program for Frege rests on a scholarly and philosophical mistake. Namely, it takes what Bertrand Russell calls the backward road from reference to sense. Since Dummett endorses the backward road, I must show that the mistake is genuine. But I need not enter the murky waters of "On Denoting" to do so, if I can make the mistake independently clear. After arguing that no senses are objects or functions, I show how we can keep Frege's context principle from bifurcating into one principle for senses and another for references.¹ I conclude by showing that intuitionism is a form of the backward road and shares in the mistake.

1. DUMMETT'S SEMANTIC PROGRAM FOR FREGE

Dummett (1995) in effect now agrees with me that there is abundant evidence that Frege actively uses the context principle in *Grundlagen*, and that the principle plays a key role in his arriving at his definition of Number. I advanced these views in my "Frege: Existence and Identity" (1979), which remains the most detailed treatment of these matters I know of.² We now also agree that the principle is implicitly in *Grundgesetze* (my 1996/2003, 279–87).

Dummett says, "The context principle is not concerned with principles of *definition*, however, but with how the *primitive* symbols of the system are to be explained: by what means should the semantic theory stipulate their *Bedeutungen* and thereby their *Sinne*?" (1995, 14–15). Dummett is

openly saying Frege's context principle requires that Frege's semantic theory take the backward road from references "and thereby" to senses. But Dummett's question is apparently rhetorical, because he then says that no stipulations should be needed, since the theory ought to explain how to derive all references, except for our knowledge of the two truth-values, from truth-conditions. Dummett interprets Frege's semantic program as follows:

I interpret [Frege] as follows. A *theory of Bedeutung* for a language must of course be stated in some language; if it is concerned to lay down the interpretation of the object-language, it must necessarily be stated in a metalanguage disjoint from the object-language, since the latter cannot yet be understood. *Now if the theory is to display the existing or intended Sinne of the expressions of the object-language, it must embody only what anyone who has a mastery of the object-language will know.* As far as possible, therefore, it must refrain from exploiting what can be known only by understanding the metalanguage. There is an inescapable exception to this. Anyone who makes a judgment knows what the two truth-values are; but this knowledge cannot be stated. Only one who can grasp a thought can make a judgment, and, among human beings, only one who can speak a language can grasp a thought; nevertheless, we can say what is known by someone who is master of a language only by presupposing the two truth-values as known.

Our theory may accordingly specify outright the conditions for a sentence *of a given form* to be true or false, that is, to have one or other truth-value as its *Bedeutung*; but it cannot fix the *Bedeutung* of any term by specifying that it is to be any other object namable in the metalanguage. *A mastery of the object-language consists in grasping the condition for each of its sentences to be true. It is only in doing so that one who knows that language knows the Bedeutungen of its terms; hence to stipulate in the metalanguage that certain terms are to denote certain objects is at best to take as understood what the theory should be explaining.* The only way in which a speaker of the object-language can specify the *Bedeutung* of one of its terms is by using some co-referential term, *which he recognises as such by his grasp of statements of identity.* This is but part of his knowledge of what determines the truth or falsity of sentences containing the term; it is the content of that knowledge which the theory of *Bedeutung* must exhibit. (1995, 16–17, emphasis mine)

In this puzzling passage, it looks for all the world as if Dummett is saying that it is by specifying the truth-conditions of a "sentence of a given form," and "only [by] doing so," that we can explain the references of its subsentential terms, and thereby also the "intended *Sinne*" expressed by those terms. The sole exception noted is that we must presuppose knowledge of what the truth-values are, though not of which sentences are true and which are false. These objects are Frege's sentential references. But they are also subsentential references. We can use "the True" to refer to the

True. It might seem from the phrase “any *other* object [than a truth-value]” that Dummett is allowing us to fix the *Bedeutung* of “the True” as the True. This seems to contradict the aim of the program. But the phrase’s grammatical reference is unclear to me, and I shall charitably assume Dummett does not mean that. In any case, the passage states the semantic program Dummett interprets Frege as having. I shall call it “Dummett’s program.”

Dummett is saying that for Frege, there is a backward road from reference to sense. This departs from Dummett’s earlier books in which he held Russell is right that there is no backward road for Frege (1981a, 87, 95; 1981, 267), and amended Frege’s theory because the no backward road thesis is a “problem” (1981a, 87–88, citing 1981, 268). My view is that the earlier Dummett is right that for Frege there is no backward road, and the 1993 Dummett is right that the context principle is actively used in *Grundlagen* and implicit in *Grundgesetze*. But the 1993 Dummett is interpreting the context principle too strongly, so that it permits and even requires a backward road in Frege’s theory of sense and reference.

2. PROBLEMS WITH THE PROGRAM

Dummett’s program has at least seven problems.

The first problem is a dilemma based on an ambiguity. Dummett says, “Our theory may accordingly specify outright the conditions for a sentence *of a given form* to be true or false. . . .” But when we are given the form of a sentence, does he mean that we are given (1) *that* the form of sentence S *is* the form of, say, an identity statement? If so, then we are not given *which* subsentential name in S refers to the identity relation or *which* subsentential names in S are the logical subject names. Even if S is “ $a = b$,” we have no right to assume that the subsentential expressions are what they usually are. How can we derive references and senses for subsentential names if we do not even know how to parse a sentence into names, that is, if we do not know which physical portions of the sentence to count as names? We would be lost before we started. We have only a list of complete sentences of the object language to work with, and not a list of subsentential names. Or does he mean that (2) we are given (1) *plus* which portion of an identity statement to count as the name referring to the identity relation and which portions to count as the logical subject names?

It might seem that Dummett has option (1) in mind, since he continues, “that is, to have one or other truth-value as its *Bedeutung*,” which would not give us any clue as to how to parse sentence S into names. And he further continues, “but it cannot fix the *Bedeutung* of any term by specifying that

it is to be any other object namable in the metalanguage.” This seems to rule out option (2) expressly. But if so, it is puzzlingly worded. For the *Bedeutung* of the term for identity is not an object but a relation. In fact, every sentence must have at least one term, namely its logical predicate, whose *Bedeutung* cannot be an object. Surely Dummett’s program aims to derive references and senses for all primitive names, not just for primitive object names.

On option (2), Dummett would be illicitly appealing to our prior understanding of the term for identity. That term is not only subsentential but primitive, and he has ruled out our prior understanding of precisely such terms as “to take as understood what the theory should be explaining.” But if he does take option (2), his motive is obvious. As Dummett knows, grasping “statements of identity” is key to specifying the reference of a subsentential expression (1995, 17). We cannot know which two terms in an identity statement are the co-referential ones, i.e., the logical subject names, unless we also know which term is the identity name. And the name for identity is one of the primitive names the reference of which Dummett’s program is trying to derive. The whole point of the program is to show how to derive references “and thereby” senses for all primitive names from truth-conditions alone.

Frege admits eight primitive logical names, including that of identity, in *Grundgesetze* vol. 1, §31. There Frege proves that all eight primitive logical names refer. But how do we come to grasp their senses? Does Frege state any procedure for deriving or grasping their senses in §32? That is, do we look to statements in which primitive names occur, treat the thoughts the statements express as the thoughts that the statements’ truth-conditions are fulfilled, then logically derive the references of the primitive names, and then identify the senses of the primitive names as the contributions those names make to the expression of this thought? Do we, perhaps, look to the truth-conditions for identity statements *about* identity³ and the other primitive logical entities in order to determine which names refer to them and thereby the names’ senses?

Absolutely not. That would be a magical procedure, like pulling a rabbit out of a hat. For Quinean problems of inscrutability of reference and indeterminacy of translation implicitly abound in Frege as early as 1884 in *Grundlagen* §22, where one card pack is also many cards, and as late as the permutation of red and green in “The Thought.” Dummett says, “In *Grundgesetze*, Volume I, §10 . . . Frege uses a permutation argument to show that the *Bedeutungen* of value-range terms have not yet been fixed. This appears to imply that he had ceased to believe that questions of reference are internal to language” (1995, 13). Indeed, the card pack and

duality of geometry permutation arguments in *Grundlagen* appear to imply that Frege did not believe that questions of reference are internal to language as early as 1884.⁴ All this applies to purely logical names as well. Thus the sense and reference of the identity name must be assumed prior to any derivations of sense and reference for any other subsentential names—purely logical or otherwise—in the object language (Frege's formal notation) from the truth-conditions of statements in the object language. Otherwise we beg the question of the identity of identity. Similarly for all the other primitive logical names. I think Dummett knows this. And I think that is why he is smuggling in our prior knowledge of the references and senses of all eight of Frege's primitive logical names under the guise of their belonging to the logical form of statements. But their belonging to logical form scarcely hides their being subsentential names which express senses and refer to references. Elsewhere, Dummett says that the context principle “must apply to the logical constants as much as to other words” (2000, 252). Thus I think he must have option (2) in mind after all.

The second problem is that of referential inscrutability of nonlogical terms. Even on option (2), i.e., already fixing the senses and references of all eight primitive logical names in the object language, which eliminates the permutation problem for those eight names, and for all complex logical names defined in terms of them, the permutation problem remains for all nonlogical names, such as names of card packs and of figures subject to dual geometrical theorems. That is, we cannot transcendently deduce the world of references from pure logic plus truth-conditions alone. For primitive logical names are not the only primitive names. And even within pure logic, as we just saw, Dummett admits that in *Grundgesetze*, Frege uses a permutation argument to show that value-range terms do not yet have references. And that is with the eight primitive logical names' already expressing their senses and referring to their references.

For Frege and Quine, having a referential apparatus (“logical form”) alone, even including identity, is not enough to preclude systematic permutations. That is why Frege would require explications of some nonlogical terms, and why Quine requires references to rabbits in a home language. Quine says it is not the logical identity term, but sortal terms such as “rabbit,” that do the real work of individuation (1981, 12; 1975, 91, 116). Surely Frege would agree, using sortal concepts such as *card pack*. I call this the permutation problem for references.

More deeply, the problem is this. The truth-conditions of identity statements cannot be understood unless we already understand (the identity conditions of) the objects they are about. These are so interdependent, they are distinct only in reason. This might seem to give Dummett's program at

least a Pyrrhic victory. But Frege starts with explications of names as opposed to statements. Thus for Frege, Dummett's program presupposes what it purports to derive.

Third, senses have the same problem. Frege says, "the thought itself does not yet determine what is to be regarded as the subject" (1971d, 49; see 46 n.*). Thus, since "a thought can be split up in many ways," so that many things can appear as the subject (1971d, 49; see 46 n.*), there is a permutation problem even for the *senses* of logical subject names. I call this the permutation problem for senses, or the problem of sensial inscrutability.

The fourth problem is that as Dummett says, "according to Frege, the sense of an expression determines its reference" (1981, 266). He even says that "sense determines reference but not conversely" (1981, 274). This is the forward road from sense to reference. But on Dummett's program, reference determines sense. This is a problem of circularity.

The fifth problem is that the senses of the primitive logical names are underdetermined by their references. Thus even if Dummett's program magically succeeded in deriving the references of all subsentential names from the truth-conditions, each reference can be presented in indefinitely many ways. Even the identity relation can be presented to us in many ways. Thus it is magical to suppose that we can derive intensional senses from truth-conditions, which consist of extensional references, even if we assign truth-conditions to all statements in the object language. I call this the problem of sensial underdetermination.⁵

Whitehead and Russell distinguish four classic senses of the word "intension." (1) There are propositional functions that are not truth-functional, e.g., "*A* believes that *p*" (1950, 8; see 187 for a derivative sense of "intensional proposition"). (2) There are propositional functions that lack extensional identities—"the same class of objects will have many determining functions" (1950, 23). Such functions are called formally equivalent (1950, 21, 72–73). We may say more generally that different ways of presenting a thing are intensional in this sense. (3) There are intensional functions in the sense that their values need not be specified for them to be specified (1950, 39–40). This sense is inimical to intuitionism. (4) Where extensional classes ("extensions") are identical just in case their members are identical (1950, *20.31, *20.43), by implication a class is intensional if it is not extensional. (Whitehead and Russell use only extensional classes.)

There are four corresponding senses of "extension." For Frege, senses (2) and (4) are logically tied. That is because for him, all functions are extensional in sense (2) and all classes are extensional in sense (4). Functions correspond one-one with their courses-of-values, where a course-of-values is the class of ordered pairs of arguments and values mapped by

the function. For a function and its course-of-values represent each other via the representation function. The representation function is formally well-defined as mutual and is therefore one-one (Frege 1964, 92–94). We may say that functions which have the same course-of-values are representatively identical. Representatively identical functions are not different functions, though due to the “peculiarity” of predicative language we cannot directly say so (Frege 1971d, 46). Since functions are incomplete, they cannot directly stand in the identity relation. A function cannot even be directly said to be identical with itself. For “in view of its predicative nature, it must first be . . . represented by an object” (Frege 1971d, 46). It is the representing object that stands in the identity relation. We representatively say that functions are identical when we say that their representing objects are identical. Since mutual representation would be impossible if there were not a one-one correspondence between functions and their courses-of-values, the representative identity conditions of functions are exactly as sharp as the identity conditions of the objects that represent them. That this one-one correspondence obtains is the famous extensionality thesis (Furth 1964, xl–xliv), whose name we may honor by saying that for Frege, functions are always extensional in sense (2). A function is definable as any equivalent function (Frege 1971c, 80). Functions are equivalent if and only if their courses-of-values are identical (Frege 1964, 43–44). Thus equivalence is the relation of representative identity. The equivalence relation and the identity relation represent each other, as do their respective relata. But the identity relation itself is indefinable, for a technical reason: since a definition is a stipulated identity, all definitions presuppose identity (Frege, 1971c, 80–81). More precisely, a definition of a function is a stipulated representative identity.

When Dummett contrasts intensionality with truth-functionality, he has sense (1) implicitly in mind. When he discusses senses as different ways of presenting one object, he has sense (2) implicitly in mind. And when he discusses our inability to traverse infinitely many objects which might fall under a concept, he has sense (3) implicitly in mind. But I cannot recall that he ever expressly distinguishes these three *Principia* senses of “intension.” I doubt even more that he distinguishes the corresponding three *Principia* senses of “extension.” In his Frege books, he introduces a single notion of extension in terms of both truth-functionality and opacity, blurring senses (1) and (2). The slip from sense (1) to sense (2) is subtle:

. . . are . . . extensional: that is, provided that the truth-value of a complex sentence depends only on the truth-values of the constituents. . . . What happens, then, in a case in which a method of sentence-formation is used which is *not extensional*, that is, when a sentence is formed which contains an opaque context? (Dummett 1981, 189–90, emphasis mine; see 1981a, 155)

But he does clearly identify intension in sense (2) in discussing intuitionism (2000, 16–17).

For Frege, all sentential contexts, even opaque contexts, are extensional in sense (1). To preserve truth-functionality in opaque contexts, Frege introduces a systematic reference shift from customary references to senses. Thus senses are extensional in sense (1). But senses are intensional in sense (2). That is the whole point of Frege's explanation of informative identity statements. Namely, different names expressing different senses can refer to the same reference.

All senses are intensional in sense (2), while all customary, i.e., direct, references are extensional in sense (2). For any customary reference, there are indefinitely many senses containing modes of presentation of that reference. Insofar as a reference is informatively identifiable indefinitely many times, there are indefinitely many senses through which it can be identified. Thus in general, the relation of a reference to the senses which are ways of identifying it is one-many. Thus it is magical to suppose that starting from any given set of references, i.e., objects and functions, we can work back so as to determine any one sense in particular as the sense expressed by a name of any one of those references in a given sentence.

The same goes for truth-conditions. For a truth-condition consists of extensional in sense (2) references. It is the extensional in sense (2) references of the subsentential names, in some extensional in sense (2) order, which make a customary statement true or false.

We must not beg the question by using descriptions of truth-conditions to identify them prior to the promised derivation of subsentential references and thereby subsentential senses from them. For such descriptions are intensional in sense (2), while the truth-conditions themselves are extensional in sense (2). Truth-conditions do not contain modes of presentation of truth-values. Thoughts do. Truth-conditions are not intensional in sense (2) because they are not ways of presenting or identifying anything. They are presented or identified in different ways by thoughts. I think Dummett would agree with this (1981, 5, 634, 642; 1981a, 252–53).

The sixth problem is that Dummett's program discusses only Frege's semantic stage of stipulative regimentation, and entirely ignores Frege's initial semantic stage of explication.

How can we convey the senses of primitive names? Is it by stipulation, as I quoted Dummett as saying? No. Stipulation has all the advantage of theft over honest explicative toil, as Russell might say. For Frege, stipulations cannot be made for primitive names except in the sense of stipulated regimentations of reference to ensure logical determinacy. Regimentations can be highly artificial. But if a regimentation of a primitive

name which already has a sense is intuitively correct—notably, regimenting the identity name so that an object is truly said to be identical with itself but not identical with anything else (*Grundgesetze* vol. 1, §7)—Frege calls it an *analysis* (1979f, 210). Frege's only other stipulations are his definitions. For Frege, all definitions are stipulations which fix sense and reference for the defined names (*Grundgesetze* vol. 1, §27; 1979f, 207–8, 210, 211), and as stipulations, they are neither true nor false (1980c, 36; 1971f, 24; 1971e, 50–51). Of course, explications are not true or false either. But there is an important sense in which stipulations logically cannot fail, and are thus scientifically certain. Namely, they are stipulations! This is very different from explications.

Frege's view in *Grundgesetze* is that we cannot figure out subsentential references unless we already grasp as primitive the apparatus of identity and quantification, much as in Quine, and also negation, material implication, and some other functions. In particular, Frege's stipulation of logical determinacy for the identity relation in vol. 1, §7 is not a definition, since Frege lists " $\xi = \zeta$ " as a primitive name in vol. 1, §31, and argues elsewhere that identity is indefinable because every definition is an identity (1971c, 80). Thus §7 can only be a regimentation of identity merely to ensure its logical determinacy. Frege says he has been talking about identity all along, i.e., prior to its regimentation (*Grundgesetze* vol. 1, §7; compare 1979f, 210).

Some terms need less regimentation than others, since some explications are more determinate than others to begin with. It is a short jump from the ordinary sense of "gleich" to the regimented sense of the formal identity sign. The case is the same for negation. But notoriously, truth-functional "or" is a big regimentation. And the horizontal stroke is a big regimentation that prevents any well-formed sentence from lacking a truth-value for categorial reasons. For example, "5 is pink" is arguably an indeterminate statement, but prefixing the horizontal stroke imposes falsehood on it.

For Frege, explications (*Erläuterungen*) are not stipulative definitions, or even stipulated regimentations. Explications are hints. Frege says right in *Grundgesetze* vol. 1, §0 (last para.) that he must indicate what he intends by his primitive names by "hints," since not everything can be defined. For Frege, communication *always* begins with hints, metaphors, or suggestions, and thus with an element of uncertainty.⁶ Thus explications logically can fail. This is essentially different from regimentations and definitions. This is why Frege requires explications to occur in a propaedeutic, while definitions, and perhaps also regimentations, occur within the scientific notation. Before we can start a formal notation at all, "we must be able to count on a meeting of minds."⁷ Thus explications are always prior to the

object language. Thus Dummett is wrong when he says Frege “was setting out what [the formal notation’s] formulas were to mean . . . not . . . from without but from within” (1995, 16). This is true of definition and perhaps of regimentation, but not of the key stage of explication. In fact, the meeting of minds is precisely what leads Frege to require extralinguistic, extramental objective entities, so that the minds have something they can agree or disagree about in his private language arguments, though of course those arguments are scarcely limited to cases of explication.

I suspect that regimentation occurs outside the object language as well. For regimentation is a process, and only its end results, primitive names which are “proved” to have references, are permitted into the formal notation. And unlike definition, Frege has no sign for regimentation in the formal notation, no way of indicating within the notation that he is regimenting anything. On the other hand, regimentations have the same stipulative certainty as definitions, and can occur in the formal notation as safely as definitions do. But it does not matter for us here, since either way Dummett’s program omits the whole stage of explication.

Some explications must be not only prescientific but prelinguistic, on pain of vicious regress of metalanguages (compare Dummett 1981, 231). At least some hints, if not metaphors or verbal suggestions, must be prior to language altogether for first-time language learners. The very first hinter and hintee are both outside language until their minds meet. Thus not every propaedeutic can be a metalanguage. We all start language as outsiders, *pace* Dummett (1995, 16; see 10–11). There are no born insiders. Insiders and outsiders alike must *ultimately* start teaching or learning a language by explications from outside the language. Adults often use a metalanguage to learn a new language by translation, but for Frege they must always start from prelinguistic explications at some point. They can even follow the path of children, starting as outsiders by taking a course of prelinguistic explications directly of the language they wish to learn.⁸

For Frege, references are conveyed only via senses. Frege says “it is *via* a sense, and only *via* a sense that a proper name is related to an object” (1979c, 124). Thus explications convey references only via senses. Thus every explication is first and foremost an explication of a sense.

Frege says that the regimentation process in §31 “also” fixes senses:

Not only a denotation [i.e. reference], but also a sense appertains to all names

correctly formed from our signs. Every such name of a truth-value *expresses* a sense, a *thought*. Namely, by our stipulations it is determined under what conditions the name denotes the True. The sense of this name—the thought—is the thought that these conditions are fulfilled. (*Grundgesetze* vol. 1, §32)

This is the text Dummett bases his program on. But Frege says references and *also* senses, not references and *thereby* senses. That fits his primary focus on proving references for the names. And we cannot stipulate (regiment) any primitive names until we first explicate them, on pain of there being nothing to regiment. And every explication is first and foremost a conveyance of a sense. Thus there is no transcendental deduction of senses from references in the regimentation process. It is merely unregimented senses in, regimented senses out, not magical, but sensible. There is no reversal of Frege's thesis that references are conveyed only via senses. When compounded into thoughts, the explicated senses present the truth-conditions, not the other way around. Regimentation merely ensures their logical determinacy.

Thus the order of logical priority for Frege is: (1) explications of the senses and thereby the references of the primitive names; then (2) regimentations of the senses and thereby the references of the primitive names into logical determinacy, "proving" that they have references, and regimenting thoughts and thereby truth-conditions in the process, thus satisfying the context principle for senses and thereby for references; then (3) definitions of any defined names. §32 implies no backward procedure for identifying senses given the truth-conditions of statements. On its face, it is merely a general statement of what senses essentially do and thereby are.

Dummett seems to think Frege's regimentation of primitive names fixes truth-conditions first, then subsentential references and thereby subsentential senses. This is backwards. He might as well say Frege's thesis is that senses are conveyed only via references. But if we regiment unexplicated blank names, we obtain blank truth-conditions. *Ex nihilo, nihil fit*. We really need that first stage of explication!

Frege's context principle can never be interpreted so strongly as to destroy Frege's theory of language as beginning with explication. And it would make a mockery of explication as the start of language learning to suggest that it must be done in accordance with an already assumed list of the truth-conditions for any sentences. I do not see how we could have even the concept of a truth-condition, much less the concept of a stipulation, prior to the use of explication to get language started by conveying some undefined senses and thereby some undefined references. In the lexicon of Wittgenstein, explication is Frege's rock bottom. It is what we do to start language at all. Dummett discusses explication elsewhere, but his program ignores it completely.

We can, of course, explicate a name in the metalanguage, e.g., "gleich," and then stipulate that a certain sign in the object language—say "="—is to express the same sense as the explicated metalinguistic name. Perhaps that is what Dummett has in mind. But this only postpones the problem to the

metalanguage. On pain of vicious regress of metalanguages, some names for Frege's eight primitive references must be explicated before we can stipulate that any names in the object language refer to those references. Thus even if Dummett wins the battle on the object language, he loses the war on language in general.

The seventh problem is that since references are conveyed only via senses, regimentation regimentations references only via senses. Thus regimentation is first and foremost regimentation of senses (compare Dummett 1981, 626). And for Frege, everything is regimented at once. This resolves the problem of double *oratio obliqua* (in ordinary language, the *obliqua* chips fall where they may). Since all opaque contexts and all senses, however indirect, are regimented at once, it is always determinate which sense is expressed in which context. But if we start with blank terms, each will take in all other's washing.

Definitions stipulate references only via senses, but Dummett's program rightly does not concern them. For Frege, definitions are never needed to say what we want to say. They merely abbreviate.

3. DOES THE PROGRAM WORK?

Dummett never attempts an example of how his program works. I have already argued that it does not work even if we pump it up with advance knowledge of which portions of sentences to count as subsentential names, and of the senses and references of all primitive logical names, in the object language. But an example may help.

Suppose the vocabulary of object language L is already given as four blank object names, "*a*," "*b*," "*c*," and "*d*," one blank concept-name, "*F*()," "blank" meaning without their senses or references, plus Frege's eight primitive logical names with their senses and references. We are then told the truth-conditions of all atomic sentences of L. That can be done only via thoughts, but we can then disregard the thoughts in a sleight of hand in place of the magical start the program demands. Now, if we knew which objects in the world were *a*, *b*, *c*, and *d*, and that they were the only four objects in the world, and (3) that only *a*, *b*, and *c* fall under the concept *F*, then there would be no permutation problems for the object names (e.g., cards or decks), and we could enumeratively define "*F*()" as referring to the function that maps the arguments *a*, *b*, and *c*, but not *d* onto the True. But we are not entitled to know these things on Dummett's program. Knowing them would amount to knowing the truth-values of the sentences, and we were only supposed to know their truth-conditions.

But let us pretend that we magically succeeded in defining the reference

of " $F()$ " in the enumerative manner suggested. Can we thereby derive the sense expressed by " $F()$ " as the same as that expressed by " $()$ falls under the purely extensional in sense (2) mapping-function which maps the arguments a , b , and c , but not d onto the value the True"? I cannot think of what else Dummett might have in mind. But the reference of " $F()$ " can be presented in indefinitely many ways, for example, as the reference now under discussion. Thus there can be indefinitely many object languages L_1 . . . L_n that agree on the reference but differ on the sense of " $F()$."

Suppose that all and only a , b , and c are red and round. How can we tell whether speakers of L are using " $F()$ " to express the sense ' $()$ is red', ' $()$ is round', ' $()$ is red and round', or ' $()$ is red or round'? Or in a world in which all and only red objects are round, and in which infinitely many objects are red, so that $F()$ must be intensional in sense (3), how can we identify the sense expressed by " $F()$ "? By hypothesis, the only senses " $F()$ " could possibly express for us are: ' $()$ is red', ' $()$ is round', ' $()$ is red and round', or ' $()$ is red or round'. Which sense is it? How can we identify which sense it is without already grasping the senses expressed by "red" or "round"? Since a is an object and concept F is merely a mapping function, the truth-condition, Fa , is purely extensional in sense (2), and leaves us clueless as to which sense is expressed by " Fa ." What kind of ideal language is it in which we cannot even distinguish colors from shapes?

Is L perhaps too small? It seems large enough to refute Dummett's program. And we can always add more names.

Appealing to "All red things have color" or "All round things have shape" will not do. Where all and only red things are round, these two statements have the same extensional in sense (2) truth-condition. We cannot even say "Red is a color" in Frege's notation. The closest we can come is "If anything is red, then it has a color." And L has only one nonlogical predicate! But it would not help even if we could say it, since there would be a Quinean compensatory adjustment of "color" and "shape" as well.

Are not the concepts *red* and *round* differently principled and therefore different mapping functions? The premise is true but the conclusion does not follow. They are differently principled by the senses expressed by "red" and "round." More precisely, incomplete senses contain modes of presentation which are the mapping principles of functions.

Dummett's program fails even if we provide truth-conditions for all possible sentences of L as used to describe all possible truth conditions. Even if such a procedure were correct in principle, it would be unworkable in practice. No human can learn that "is red" expresses the sense of a color by working through all possible sentences of English as used to describe all possible truth conditions. Dummett would be the first to tell us that we

cannot even work through all actually used sentences of English, but must be able to use only a finite fragment of a language to learn senses and references. Yet his view implies just the opposite:

Suppose, then, that we have two sentences which are analytically equivalent, but have different senses. Since they have different senses, they must, on a view of Frege's kind, have different truth-conditions. Since they are analytically equivalent, the world cannot be so constituted that one is true and the other false: the set of possible worlds in which the one is true is the very same set as that of those in which the other is true. How can there be room for Frege to distinguish the truth-conditions of the one from those of the other? (1981, 588)

Here Dummett adduces a nonexistent problem from a strange view of Frege's truth-conditions. If Dummett thinks that difference in sense implies difference in truth-conditions, then perhaps that is why he thinks his program will work! For then sameness of truth-condition implies sameness of sense, which is the backward road. The implication is really that truth-conditions are as intensional in sense (2) as senses are. But does that make any sense? Do not truth-conditions consist of references? Are then all customary references intensional in sense (2)? Or can we truth-functionally compose intensional in sense (2) truth-conditions out of extensional in sense (2) customary references? Or perhaps the implication is that senses are as *extensional* in sense (2) as truth-conditions are!

One might suggest that all analytic truths have the same truth-condition, for example, " $2 + 2 = 4$ " and " $(x)(x = x)$." Is that not so? Their truth-condition is true under any condition! Would not Frege agree (1979d, 187–88; 1979f, 208 on $x = x$)? *Pace* Dummett (1995, 12), Frege adopts a theory of logical necessity as purely general truth, i.e., as invariance of interpretation, and holds it for at least twenty-seven years, from 1879 *Begriffsschrift*, §4 to 1906 "Introduction to Logic" (1979d, 187, 188, 189). Frege even uses the word "vary" twice (1979d, 187, 188).¹⁰

On this suggestion, if Frege really makes logical equivalence the criterion for the identity of thoughts (1971d, 46 n.*), *contra* Dummett's view the test is their *given* identity (1981, 632)—Dummett's test contradicts the last two sentences of Frege's note n.*—this would provide only limited help to Dummett's program. Yes, all logical truths would then slice up the same thought and slice up the same truth-condition, true under any condition. Then in a formal notation consisting only of logical truths, we could always derive one and the same thought from one and the same truth-condition, no matter what the logical truth. The case is the same for logical falsehoods, and for all sets of statements which are true in exactly the same

possible worlds. But the problems of referential and sensial inscrutability of subsentential names would remain. And Frege rejects possible worlds. For him, there is no such thing as a merely possible object, *Grundgesetze* (1971a, 222). As Dummett puts it perfectly, Frege's intended model is the actual world (1995, 12 n.5). And in the actual world there are clearly many pairs of logically nonequivalent predicates expressing different senses yet referring to the same extensional in sense (2) mapping function. Our world of four objects logically could be the actual world. Thus even if logical equivalence were Frege's criterion for the identity of thoughts, the problem of sensial underdetermination would remain for indefinitely many logically contingent statements in an ideal scientific language (compare Dummett 1981, 228, 636).

But I reject the suggestion and I accept Dummett's view that sensial identity is given. Perhaps all analytic truths have the same *general* truth-condition, true under any condition. But even if necessary truth is the same as purely general truth for Frege, the *specific* or *proper* truth-conditions of " $2 + 2 = 4$ " and " $(x)(x = x)$ " are different, consisting of different specific functions and objects in different specific relationships. Strictly, even " $2 + 2 = 4$ " and " $4 - 2 = 2$ " have different truth-conditions and express different thoughts. As to sensial identity, Frege's plea that we must allow transformations of the same thought in logical inference can be substantially preserved by glossing transformations as changes to new thoughts which are substantially the same and differ only holistically in their slicing. But on pain of vicious cognitive regress, it is essential that all senses *logically can* be directly given, no matter how indirect the sense. Dummett is absolutely right that given sensial identity is fundamental and trumps transformations (1981, 631–37). But then even sameness of truth-condition does not imply sameness of sense, since Frege's specific truth-conditions are extensional in sense (2), exactly like the customary references they consist of, while his given senses are intensional in sense (2). This is why Frege would reject even a definition of a predicate's intension as the class of all possible objects satisfying it (compare C. I. Lewis). For on that definition, the senses of " $= (1 + 1)$ " and " $= (4 - 2)$ " are the same.

The actual world is only the actual set of *compossible* truth-conditions. But even if we knew all possible truth-conditions as composed of all possible customary references in all possible worlds, why assume that the problem of sensial underdetermination would somehow magically vanish? Even if to give all possible customary references is to give all their possible modes of presentation via each other (I waive the possibility of presentations of customary references via their relationships to names, senses, forces, tones, and ideas), where is the one-one correlation between

all possible customary references and all possible customary senses?

Even assigning all the truth-values of all possible sentences of L would leave us clueless, since truth-values are extensional in sense (2), just as truth-conditions are, and there is no backward road. For example, add all the truth-values of all the atomic sentences to my example. Namely, “*Fa*,” “*Fb*,” and “*Fc*” refer to the True and “*Fd*” refers to the False. How could that help? We simply cannot squeeze any intensions in sense (2) out of extensions in sense (2), even if we include all truth-value assignments as well as all truth-conditions among the extensions. There is always sensial underdetermination. This intimates the consequences for intuitionism.

We must distinguish the truth-condition of a statement from a statement of the truth-condition. The former is extensional in sense (2). The latter is intensional in sense (2), even if it describes a definitional enumeration of a mere set by its members. And it expresses a thought whose component senses, intensional in sense (2), refer to the extensional in sense (2) references which compose the extensional in sense (2) truth-condition (compare Dummett 1981a, 80). This might be called a logically indirect relationship in virtue of the mediating roles of the component senses and component references. Frege’s semantic schema is not truth-describes-fact, but function-maps-value-onto-argument(s).¹¹ An extensional in sense (2) truth-condition is just that a certain concept does map its argument(s) onto the True as asserted. But even the previous sentence expresses an intensional thought in sense (2). And although that thought reflects or mirrors the essence of concepts (my 2003, xix, 31, 217, 272, 301), it is only one way of many to present the truth-condition in question. Thus there is no backward road from the truth-condition to it.

Certainly we do not understand the thoughts expressed by statements if we have no idea of the conditions which would make them true or false. But those very thoughts of the truth-conditions are intensional in sense (2), and the truth-conditions are graspable only via such thoughts. Indeed, the thought we grasp *is* the “idea” we have of the truth-condition.

One might be tempted to say that where “*F*()” expresses the sense ‘() is red’, the truth condition of “*Fa*” is *a*’s being red. That is what most of us would say. But can Frege say it? Our world of four objects shows that for Frege, the ontological locus of objective redness as opposed to objective roundness is the realm of sense. And as this is a categorial matter this is so in every possible world. Is the ontological locus of the truth-condition of “*Fa*” then the realm of sense? Is it senses, not references, which make our sentences true? Except for cases of indirect reference, no.¹²

If we enrich L so as to include sentences expressing propositional attitudes, then thoughts would be logical parts of the truth-conditions of L,

since thoughts are the indirect references of such sentences for Frege. And while only thoughts can be believed, and subsentential senses cannot be believed, presumably the subsentential senses composing believed thoughts must also be indirect references by parity of reason. But Dummett's program would still fail for infinitely many languages L^* which lack such sentences, and he gives no indication that he is thinking of this limited way out. And there is no backward road to the indirect senses which occur in cases of indirect reference. We would have to include sentences expressing second-order beliefs about beliefs so as to make first-order indirect senses into second-order indirect references, and so on for any n -level order of beliefs. Thus we would have to admit an infinite series of orders of indirect sense and reference. I think Frege allows and requires that (Dummett agrees, 1981a, 87).¹³ But Dummett's program would fail for infinitely many more languages L^{**} , which lack various higher orders of propositional attitudes, and again, Dummett gives no indication that he is thinking of this limited way out.

That senses are truth-functional, i.e., extensional in sense (1), does not make a backward road possible. To the contrary, it is precisely because senses are truth-functional that the problem of permuting the obviously different senses expressed by "red()" and "round()" arises *within* the confines of truth-functionality. For Frege, the problem of opacity arises, and can be solved by a reference shift in opaque contexts, *because* things are presented in different ways. That things are presented in different ways is not a problem but the pre-philosophical truth grounding the sense-reference distinction, implying the "no backward road" thesis as its *alter ego*, and underlying both the problem of opacity and Frege's solution. This is why senses (1) and (2) of "extensional" must not be blurred. It is also why Dummett's emendation of accepting Frege's shift of reference from customary reference to customary sense (indirect reference) in opaque contexts, but omitting Frege's shift of sense from customary sense to indirect sense, logically concerns only the problem of *oratio obliqua*, and is too shallow to touch the general "problem" of no backward road, *pace* Dummett (1981a, 87). For things are presented to us in different ways even if our language has no opaque contexts.

Of course, Dummett could insist on his interpretation and say the problems are Frege's. But my interpretation avoids the problems. Frege can easily construct L by explicating his eight primitive logical names, explicating " a ," " b ," " c ," and " d " as primitive names of concrete objects, and by either explicating " $F()$ " as expressing a primitive sense, say, that of "red," or defining " $F()$ " as expressing a complex sense, say, that of "red and round," or defining " $F()$ " extensionally as the concept under which a ,

b, and *c*, but not *d*, fall. On any of these options we ensure the determinacy of all names by ensuring that every statement in which they occur has a determinate truth-value. Since we introduced only names which can occur in sentences, and since we ensured that all statements of L have a determinate truth-value, the context principle is satisfied for senses and references alike. The only thing that even appears to resemble Dummett's program is the option—I say again, *option*—of defining “*F*()” extensionally. But even this would be impossible unless the senses of the object names were explicated first.

How could a language come into being at all? Could it be by our first collecting the truth-conditions for all or some of the statements and grasping the two truth-values, and then deriving the references and thereby the senses expressed by subsentential expressions?

I think Frege would hold that children learn the senses of subsentential expressions of their language not by being handed two truth-values plus the bundle of truth-conditions for all or some sentences of that language and being told to go figure out all the subsentential references and thereby the subsentential senses (they would not even understand what they were handed, much less the instruction), but by being given prelinguistic explications of names of perceived objects, followed by a process of comparing those objects and abstracting imperceptible concept-senses and concepts.¹⁴ And sense-perception is intensional in sense (2). Of course, the children would not use these senses *as* linguistic senses at first, and in that functional sense, these senses would not yet *be* linguistic senses, i.e., they would not yet be expressed by any names. They would still be mere modes of cognitive presentation, and would become linguistic senses containing modes of presentation only when used in language according to the context principle. I think that is the essential role of the principle, and the difference Frege is concerned to distinguish is between a sense and the mode of presentation it contains, *pace* Dummett (1981, 227).¹⁵

The problems with Dummett's program seem so obvious and so many that it is natural to wonder if I understood it correctly. In particular, I am criticizing what might be called the pure, ideal, or radical backward road, and insisting that Dummett adhere rigorously to it. But Dummett does not take the pure backward road. He says we are to take that road only “as far as possible” (1995, 16). He assumes knowledge of the truth-values. I concede that this objection is a fair one. Pure or ideal theories often have problems which more subtle “compromise” programs avoid. Nonetheless, I am criticizing what is surely Dummett's ideal theory, which he is trying to approach as best as he thinks we logically can. And criticizing ideal theories can be illuminating of radical problems.

4. THE BACKWARD ROAD

Russell says “there is no backward road” from reference to sense (1971, 50). There has been no consensus on exactly what Russell’s main argument against a meaning-denotation distinction is.¹⁶ Only its conclusion is clear: generally speaking, whenever we try to refer to a sense, we succeed only in referring to a reference.¹⁷ But the “no backward road” closure clause Russell adds at the end of the argument is also clear. Dummett beautifully states what I call Russell’s explicit “no backward road” thesis as having two conjuncts: “Russell points out that, on Frege’s own principles, ‘there is no backward road’ from reference to sense: sense determines reference, but reference does not determine sense” (1981, 267; see 159). But surely Russell is implicitly raising a many-one problem in his “no backward road” thesis. Namely, the backward road requires a one-one reference-sense correspondence; but there are many ways a reference can be given. That seems the most natural reason for his thesis. In fact, it is the only reason I can think of. Why else would he hold it? Indeed, he is well aware that Frege’s many-one sense-reference correlation is precisely how Frege explains informative true identity statements (Russell 1971, 46). Thus I shall also call the thesis of sensial underdetermination Russell’s implicit “no backward road” thesis.

Russell holds that propositional attitudes are essentially not truth-functional. And as we saw, Dummett treats truth-functionality as the key to understanding extensionality and opacity. Thus for Russell and Dummett, the thesis of sensial underdetermination concerns intensions in both senses (1) and (2). But for Frege, the thesis of sensial underdetermination concerns only intensions in sense (2), since he rejects intensions in sense (1) when he makes senses truth-functional.

Dummett also beautifully states what I call Russell’s implicit “no backward road” thesis: “Since different senses may correspond to the same reference, the reference of an expression does not determine its sense” (1981a, 245). Perhaps due to his preoccupation with double *oratio obliqua*, he does not recognize this as being Russell’s implicit “no backward road” thesis, much less as being his own acceptance of that thesis. The thesis implies the second conjunct of Russell’s explicit “no backward road” thesis as Dummett states it, which conjunct in turn implies double *oratio obliqua*. In fact, the implicit thesis is a logically necessary and sufficient condition of the second conjunct of the explicit thesis. But I think the order of ontological priority is this: Different senses correspond to any given reference. Therefore no reference determines any one sense as being or containing the way that reference is given. Therefore the reference of an

expression does not determine its sense. And that fact explains the phenomena of *oratio obliqua* and double *oratio obliqua* for Frege, but it does not imply those explanations, since other explanations are available, such as Russell's theory of descriptions. Thus Dummett's emendation does not imply the denial of any of the "no backward road" theses. Things are presented in different ways even if our language has no opaque contexts at all.

If I am wrong about Russell, it remains the case that we cannot work backward from references to senses because of the sensial underdetermination problem. Thus we need not enter the murky waters of "On Denoting" in order to see why Dummett's mistake is genuine.

I think Frege would agree that there is no backward road for this very reason. Frege says that a sense "serves to illuminate only a single aspect of the reference," and that we cannot achieve "[c]omprehensive knowledge of the reference" unless we can already "say . . . whether any given sense belongs to it" (1971i, 58). Nor can we pick out an aspect, i.e., property, i.e., concept under which the reference falls, except via a sense.

In *Frege: Philosophy of Language*, Dummett admits Russell is right that there is no backward road in Frege (1981, 267–68). Nonetheless, he offers the following argument against Russell's explicit "no backward road" thesis, and by implication the implicit thesis as well:

The sense of an expression is the mode of presentation of the referent: in saying what the referent is, we have to choose a particular way of saying this, a particular means of determining something as the referent. In a case in which we are concerned to convey, or stipulate, the sense of the expression, we shall choose that means of stating what the reference is which displays the sense: we might here borrow a famous pair of terms from the *Tractatus*, and say that, for Frege, we say what the referent of a word is, and thereby show what its sense is. (This is the correct answer to Russell's objection in 'On Denoting' to Frege's theory, considered generally, rather than apropos of oblique reference, that there is 'no backward road' from reference to sense.) (1981, 227)

I call this *the forced choice argument*. Dummett is arguing as follows. A reference must be given in some way. And a sense is a way of giving a reference (if any). Therefore we can fix a sense by simply choosing it as the way we choose to fix the reference (if any). Therefore we can fix a reference and thereby fix a sense. The implied conclusion is that this removes the sensial underdetermination problem with Dummett's program. The argument seems supported by Frege: "in saying something about the meaning [i.e., reference] of the sign '3 + 5', I express a sense" (Frege 1980b, 149). But the argument gets things backwards. Dummett might as well say that Frege introduces references to explain how we can identify senses.¹⁸ That we talk about reference "only *via* a sense" (Frege 1979c, 124)

does not support Dummett's argument. Quite the opposite. If a reference must be given in some way, so that fixing a sense for an expression is a necessary condition of fixing the reference (if any) of that expression, then the logical priority is that of fixing the sense over fixing the reference. And this is the forward road from sense to reference. I can convey a reference to you via a sense of my choice only if I have already singled out the reference via some sense. And you will grasp the reference via the sense I choose, not the sense I choose via the reference. I think the (1980b, 149) text merely indicates that senses are required in addition to references in order to explain informative identity statements.¹⁹

In *Frege: Philosophy of Language*, there is a passage that seems to anticipate, and even to call for, Dummett's program as a theoretical ideal explaining Frege's "whole theory of meaning," but finding the program too problematic to succeed (1981, 652–53). The forced choice argument seems to be what carries us from references to senses (1981, 653). If the forced choice argument is Frege's, there is no need for Dummett's emendation, since the argument implies a forced choice of all senses, including indirect senses. But be that as it may, the first problem for Dummett is that Frege "failed almost completely" to provide "an account of the other means that exist, besides definition, . . . for introducing expressions into the language" (Dummett 1981, 652–53). The answer is explication; there is also explanation. (Frege is indeed brief on these matters.) The second problem for Dummett seems to be that truth-conditions do not determine a "unique solution" of subsentential references (1981, 653). This is the permutation problem for references.

Dummett says, "The references of the component expressions constitute their respective contributions to the determination of its truth-value; and the sense of any one of them constitutes the particular way in which its reference is given to one who grasps the thought" (1991, 193). He takes this from *Grundgesetze* vol. 1, §32 almost word for word, and it is perfectly correct. But it is a vast jump from this point to the thesis that a sense can be *cognitively identified* via *cognitively identifying* the reference first. Not only is the jump a logical non sequitur, but the point is not even evidence for the thesis. If you choose to present a reference in a certain way, I must grasp your way first, then via that, your reference; and you must have grasped some way first as well. In fact, the second part of the quote states the *forward* road from senses to references.

Dummett says that "the systematic theory of *Bedeutung* provided a basis for explaining in what the *Sinn* of an expression should be taken to consist, namely the way in which its *Bedeutung* is given to a competent speaker of the language" (1995, 4–5). I agree, if this is merely the explanation of what senses are in terms of how they function. I believe that

is the explanation behind the explanation in *Grundgesetze* vol. 1, §32. But Dummett's program makes it sound for all the world as if §32 implies that given the truth-conditions of L, references are *cognitively identified* (said) and senses then are *cognitively identified* derivatively (shown when references are said), or at least can be if we choose our senses right. This suggests that the following definition (not explanation) of senses is basic to Dummett's program:

[In] the sense of expression "A" = Df the way in which the reference of "A" is given to a competent speaker of the language, or the way the reference of "A" would be given to such a speaker if "A" had a reference.

This takes the backward road. Senses are mentioned only in the definiendum, and references only in the definiens. Given the reference of "A" we can define the sense of "A." But there are two problems. First, there would be no primitive senses, since all senses would be defined. This is wrong because Frege holds that not every sense can be defined. That is precisely why Frege says that primitive names—including both the senses they express and the references they refer to—need explication. And second, since reference is given "only *via* a sense" (1979c, 124), the application of this definition in particular cases would always presuppose our grasping some sense, since the reference of "A" can only be given to us via some sense in the first place. And that is circular.

On the face of it, *Grundgesetze* vol. 1, §32 describes what senses essentially are in terms of how they function in relationship to references, but without trying to define sense in terms of reference, and without suggesting that senses can be derived from references. I think the circularity of the attempted definition of sense is the underlying reason why. Thus, to apply Frege's threefold distinction among definition (*Definition*), explanation (*Erklärung*), and explication (*Erläuterung*) (see my 2003, 73), §32 may only be understood as stating Frege's explanation (not definition) of what senses are. (An explanation states what a thing is, but without technically being a definition; Frege's sole explanation is of identity as indiscernibility.) And saying that senses are the ways references are given is the explanation underlying the explanation. Thus the difference between what Dummett seems to think §32 says and what §32 actually says is as little—and as great—as the difference between definition and explanation.

In *The Interpretation of Frege's Philosophy*, Dummett revisits his *Frege: Philosophy of Language* rejection of the "no backward road" argument. He admits there is "no backward road" in "Frege's unamended theory" (1981a, 95). He does not "attempt to examine the details of Russell's argument" (1981a, 131; see my 2003, 275–77 for the details). He

is very clear on the conclusion of the argument: whenever we try to refer to a sense, we succeed only in referring to a reference (1981a, 131). But this is a red herring. The explicit “no backward road” thesis is not a premise of the argument or its conclusion, but a closure clause at the end of the argument. Yet if Dummett rejects it, he is committed to rejecting the argument as futile, since we can then backwardly single out senses after all.

Recalling the forced choice argument, Dummett says his saying-showing gloss is merely a “suggested . . . possible retort to Russell” (1981a, 131, citing 1981, 227). The retort is that even if Russell is right that we cannot refer to a sense, it may still be the case that when we “say” a reference, we thereby “show” the sense we choose to use. Again, Dummett might as well say that Frege introduces references in order to explain how we can identify senses. Surely the opposite is true: when we “say” or express a sense, we thereby “show” how to identify the reference (if any); compare Dummett (1981a, 132).

Dummett suggests that Russell’s argument is more plausible if what he really has in mind is not objects of reference but objects of apprehension. Now, Russell does begin and end “On Denoting” with early statements of his distinction between knowledge by description and knowledge by acquaintance (1971, 41, 56). But Russell’s argument against Frege concerns only denoting phrases, and knowledge by denoting phrases is precisely what Russell is distinguishing from acquaintance (1971, 56). Thus the suggestion is not what Russell has in mind. But it does reveal what Russell’s argument is logically most deeply about. It is deeply perceptive of what the argument should have been, but was not. Whether we can single out senses at all is what Russell’s argument is logically most deeply about, though Russell’s focus is on denoting, in keeping with Russell’s linguistic turn in “On Denoting.” I would rewrite Russell’s argument this way. Frege distinguishes his categories of references, senses, forces, and tones in terms of how the entities essentially function, so as to explain basic features of language. It is because modes of presentation do not function as objects of presentation that they, and thereby senses, cannot be customary references. The moment they are treated as objects of presentation, they are no longer functioning to present something else, and in that sense they *are* no longer modes of presentation (compare Dummett 1981a, 132). Thus whenever we try to single out a sense, we must single out an object instead. Frege would reply that we directly grasp only modes (or senses), but can grasp them only as modes (or senses). It is a subtle issue. My theory of qualified objects goes between the horns of the dilemma (2003, xxvi, 47, 61, 73).

Phenomenologists hold that an intentional act cannot be identified independently of the object of the act. If we equate such an act to a Fregean sense, and such an object to a Fregean object, this might seem to support the

backward road. But a Fregean sense is not an act, and a Fregean object is not a phenomenal object, but an object in itself. For Frege, an act of judgment, query, or command cannot be identified independently of its *force* and its *sense*, and there may be no object to single out.

There are two orders of determination in Frege: the order of cognition, and the order of being.

The order of cognition or cognitive dependence is the order of what must be singled out first, in order for something else to be singled out. Namely, senses must be grasped first, and references can be grasped only via senses.

The order of being or ontological dependence is the order of what must be capable of existing first, in order for something else to have an ontological function or reason for being. Namely, customary references are the primary beings and comprise what we ordinarily consider the world, and senses function merely as modes of presentation. The reference determines what all its essential modes of presentation are and what its accidental modes of presentation can be. This is why the reference cannot backwardly determine any single mode as “the” mode of its presentation.

If we remain within the realm of senses alone, or within the realm of references alone, the two orders are the same, if we cognize independent or simple beings first. But when we consider senses and references together, so as to discuss their interrelationships, the two orders must go in opposite directions as just described. The paradox is trivial. It is precisely because senses ontologically function as mere modes of presentation that they are first in the order of cognition. Thus the order of cognition takes the forward road from senses to references, and the order of being takes the backward road from references to senses.

The explicit and the implicit “no backward road” theses assert the order of cognition. *Grundgesetze* vol. 1, §32 asserts the order of being, namely, that senses *are* nothing but ways of presenting references and thereby of helping present truth-conditions. But Dummett is treating §32 as if it belongs to the order of cognition, that is, as if it inverts Russell’s “no backward road” thesis, which belongs to that order. I think this is why Dummett interprets §32 as implying and virtually saying there is a backward road. Indeed, there is a backward road implicit in §32, but it is not the backward road Russell is denying. It is in the order of being, and does not permit senses to be singled out, cognitively determined, or cognitively derived via references.

At least within the realm of senses, Dummett has it perfectly. He says that to reduce Frege to a slogan, “in the order of *explanation* the sense of a sentence is primary, but in the order of *recognition* the sense of a word is

primary.”²⁰ This beautifully states the order of being and the order of cognition respectively for that realm.

Again, I may have misunderstood Dummett's program. He sketches it very briefly, and the problems I am finding with it seem obvious. But the program as I quoted it seems quite clear, and my interpretation of it is supported by his rejection of the explicit “no backward road” thesis in his paper (1995, 14–15). If Dummett had the order of being in mind instead of the order of cognition as the *locus* of the backward road, then he should have said he was giving a “*general* explanation” of the semantics in which he is deriving the *general* notion of a sense from the *general* notion of a reference, and the *general* notion of a thought from the *general* notion of a truth-condition, per his 1981 (5). But instead he speaks of “*primitive* symbols [and] their *Bedeutungen* and . . . *Sinne*,” as if we could derive the senses of *specific* names such as “red” from *specific* truth-conditions in the order of cognition.

Dummett's thesis that all senses are ways of identifying references, if conjoined with his thesis that all senses are identified by means of truth-conditions, which consist of references, creates a vicious circle in the order of cognition. But I think his interpretation conflates the orders of cognition and being.

The two orders form an ancient pattern. After we cognitively ascend from Plato's Cave to the Sun, we can look back to see the true place of the Cave in the order of being as participatively dependent on, but also as a distorted way of apprehending, the world of forms. On this deep ontological level, I do not see Frege as departing from tradition. This deepens my 2003 thesis.

Dummett's program appears to be an idealization of a more concrete interpretation Dummett has of Frege which I shall call “Dummett's project.” I quoted Dummett's program from near the end of Dummett's 1995 paper. Dummett's project is stated by Dummett's 1995 paper as a whole, which I shall now briefly summarize in my own way.

In *Grundlagen*, a fundamental principle is always to separate the objective (mind-independent) from the subjective (mental). The admission of mind-independent entities is Frege's basic realism. Frege argues for it using a permutational private language argument (duality of geometry). Namely, we cannot communicate private ideas because they can be permuted across speakers. In *Grundlagen*, the objective world divides into objects and (if you please) functions. There they are *Inhalten* or contents, which the later Frege splits into senses and references. Thus contents are strictly neither senses nor references. But that is because they are both, only not yet differentiated. If we were to try to differentiate them, we would find

fluctuations in Frege's talk of contents in *Grundlagen*. So to speak, sometimes he has references more in mind, sometimes senses. But the sense-reference distinction would in any case occur entirely on the objective side of the house in *Grundlagen*. The later Frege gives a general theory of sense and reference. This allows him to see a second problem of realism which is entirely on the objective side of the house. Namely, there might exist only senses, but no references. The existence of senses by themselves establishes basic realism (objectivism), since they are mind-independent entities. But the second problem of realism reveals a second core notion of realism: we are realists only if we hold the things we refer to are mind-independent, as opposed to the thoughts we express, however mind-independent our thoughts may be (compare Dummett 1981a, 133).

This raises a second problem of permutation, this time on the objective side of the house. To communicate, we must avoid permutations of senses and of references just as much as we must avoid permutations of ideas. (I believe Frege's solution is explication. Dummett believes Frege's solution is translation—basically the same as Quine's solution of translation into a home language, *pace* Dummett 1995, 16, n.8; both rely on identity.)

The context principle's role remains the same even as we shift from the first problem of realism to the second, i.e., from *Grundlagen* to *Grundgesetze*, from ideas to senses. Namely, whether objects are *Grundlagen* contents or *Grundgesetze* references, we have perceptual evidence of concrete objects, but not of abstract (noncausal) objects such as numbers. The ontological role of the context principle is to establish realism for abstract objects, even though its more general semantic role is to establish that (in some sense) all terms, concrete and abstract alike, have meaning (be it content, sense, or reference) only in the context of statements. In *Grundgesetze*, this devolves to the determinacy requirement: a name refers to a reference only if it conforms to the law of excluded middle, i.e., only if every statement in which it occurs has a determinate truth-value. Making names conform by specifying truth-conditions for the statements is what I call regimentation. So far, I think we both agree.

Dummett's project is to understand what Frege is specifically doing in *Grundgesetze*. We start with Frege's primitive terms, given as ostensibly referring names, and wish to explain how Frege thinks he proves they do in fact refer, thereby establishing realism of the second kind for abstract objects. So we already know which terms are ostensibly names of identity, negation, and so on. Following *Grundgesetze*, we limit the objects of concern to truth-values and courses-of-values (the Moon is an object in the preface). We must take truth-values as understood because Frege's proof procedure, regimentation, cannot be understood if we do not. Thus for

objects, the only permutation problem of interest would be for courses-of-values. Here I think we are still in basic agreement.

If by “deriving” subsentential references from truth-conditions, Dummett merely means proving that ostensible names refer by ensuring that every statement in which they occur has a determinate truth-value, I agree this is Frege’s project. Frege’s principles of importance here are that every true or false sentence must have a denoting logical subject, and that every such statement can be sliced in different ways into a logical subject or subjects and a logical predicate. But I believe this does not amount to identifying which terms refer to which references. For we already know not only which marks are the ostensible names, but what their ostensible references are. And that is because they were explicated as expressing certain as-yet-unregimented senses prior to the regimentation. Dummett says the terms are translated, as from metalinguistic German “gleich” to the formal identity sign. This is fine if the explication of the sense is not swept under the rug. The explication would be of “gleich,” whose sense we then transfer to the formal sign. Again, I doubt that regimentation (“derivation”) of reference is internal to the formal language, but it does not matter much for me, since explication is ultimately language-external.

The problem of sensial underdetermination remains. Dummett writes as if he thinks *Grundgesetze* vol. 1, §32 states or implies the forced choice argument. But senses are *merely* regimented by regimentation. Senses are not proved, derived, or identified via proving the references of the primitive names through regimentation, because only senses can guide us on how to regiment references in the first place. This is just what the second problem of realism is: Given senses, are there references?

5. ARE SENSES OBJECTS OR FUNCTIONS?

The most direct way to derive senses from references would be to show that some senses *are* references. That would not only kill Russell’s main argument that whenever we try to refer to senses we refer instead to references, but would make the explicit and the implicit “no backward road” theses pointless as well. Dummett’s argument that senses are objects is simple and seemingly convincing. Namely, we can refer to a sense by a name such as “Fermat’s Last Theorem,” or by a definite description such as “the sense of expression ‘A’” (1981, 226–27; see 190; 1981a, 132). But I think it does not follow that senses are objects. In fact, I think that “the sense of expression ‘A’” does not refer to an object.

On its face, Frege’s discussion in “On Concept and Object” of the

singular definite article as indicating reference to an object is a discussion of references as opposed to senses (1971d, 46–47). He confines his first mention of senses to a note (1971d, 46 n.*), introducing the sense-reference distinction in the main text only after the discussion of objects and the singular definite article (1971d, 47). He then uses that distinction to explicate objects in terms of reference as opposed to sense in the same paragraph (1971d, 48). His main discussion of senses is at the end of the paper (1971d, 54–55). There is nothing here to show that “the sense of expression ‘A’” refers to an object. If anything, his opposition of reference to sense suggests the opposite.

Let us turn to “On Sense and Reference.” Frege says, “In order to speak of the sense of an expression ‘A’ one may simply use the phrase ‘the sense of the expression “A”’” (1971i, 59). Now, this is the first sentence of the second of two paragraphs discussing indirect sense and indirect reference in reported speech (1971i, 58–59). I think this sentence is not an anomalous break in the discussion of reported speech, that is, is not an odd shift of topic to customary sense and customary reference. I think this sentence is a continuous, not to say integral, part of the two-paragraph discussion of indirect sense and indirect reference. In other words, I think this sentence indicates an exception to Frege’s rule that expressions beginning with the singular definite article refer to objects. The exception is precisely for reported speech, i.e., indirect quotation, which by definition refers to senses, as opposed to direct quotation, which refers to names (1971i, 59, 65; 1980c, 149). Senses are indirectly tied to quotation because they must be garbed in language; thus for Frege, the identity of a sense is never directly given to us, but ultimately only via names, as functionally opposed to customary references.

I think this is obscured by the fact that the two paragraphs in question mainly discuss the reported speech of a speaker, while in “the sense of expression ‘A’,” ‘A’ is a piece of speech not being attributed to a speaker. But “A” would not be an expression at all unless someone could use it, and in that general sense “the sense of expression ‘A’” does report speech. There is no doubt that words are being quoted here. We often quote words without quoting a speaker. We simply surround the words with quotation marks, for example, “Fermat’s Last Theorem,” which is just an “A”.

Dummett is right that singular definite descriptions of the form “the *F*” *customarily* refer to objects, because *customarily* they logically function as object names. But if I am right, Frege classifies “the sense of expression ‘A’” as indirect quotation, and therefore it *has* no customary reference, but only an indirect reference, which is its sense. Unless Dummett has independent evidence that some senses are objects, there is nothing here to show that some senses are objects. Indeed, on my reading it follows that no

senses can be objects, since senses are essentially indirect references. By parity of reason, “the thought that A” refers to a thought as opposed to an object. The test is precisely that it “designates” a thought (Frege 1971i, 66; see 59), and the mark is precisely that we are “concerned” with thought as opposed to reference (1971i, 63, 67). At best, it could only refer to an object which represents the thought. But Frege rules out that possibility, since he holds that it refers to the thought.²¹

On Dummett’s view, Frege’s sentence about the sense of expression “A” is even odder than I have indicated. For I assume that “A” can be any expression, including function names such as “ $F()$ ” as well as object names such as “ b .” But the sense of expression “ $F()$ ” is essentially incomplete, while all objects are essentially complete. Therefore, if “the sense of expression ‘ $F()$ ’” must refer to an object, then it cannot refer to the sense expressed by “ $F()$.” On my reading, the absurdity vanishes. “The sense of expression ‘ $F()$ ’” refers to an incomplete sense, not to an object, because it makes an indirect quotation.

My assumption might be wrong. Frege’s capital “A” looks like a function name, but it marks no argument-place. But surely Frege is using “A” to range at least over all names. And even if he is not, the problem of the sense of expression “ $F()$ ” remains.

Dummett is part of a crowd. Gustav Bergmann, Rulon S. Wells, Howard Jackson, and Charles Caton all hold that senses are objects (my 2003, 65). Michael Beaney argues that they are or ought to be (1997, 28–36; see 29 n.67).

Caton argues that all senses are objects because “the sense of expression ‘A’” always refers to an object. He admits both that all objects are complete and that some senses are incomplete. He admits that this implies the formal contradiction that some objects are both complete and incomplete. He does not propose to address the problem in his paper (1968).

In *Frege: Philosophy of Language*, Dummett says, “The sense of a predicate is indeed to be considered an object” (1981, 294). For Dummett, the only other option is that it is a function, which he rejects (1981, 291, *pace* 442). He then suggests that it is “incomplete only in that it would be necessary” to grasp it as the sense of an incomplete expression (1981, 291). That is, he thinks all senses are really complete.

I think this way out makes nonsense of the metaphysical gap-and-binding Frege requires to unify the thought as an entity. Frege says he cannot explain the unity of a thought without admitting unsaturated senses (1971d, 54), as Dummett is well aware (1981, 265). And if unsaturated senses are not really unsaturated, they cannot really unify thoughts. On the face of it, Frege’s texts expressly make saturation a transcategorical bond applying in the realm of references and the realm of senses alike. Frege’s

arguments and his language are equally serious in both realms.²² Frege says that if we allow the same thing to “occur now as object, now as concept . . . the difficulty . . . is only shifted” because *thoughts* need to “hold together” (1971d, 54; see 55), implying that functions and incomplete senses face the very same metaphysical difficulty. Indeed, he says, “The words ‘unsaturated’ and ‘predicative’ seem *more* suited to the sense than the meaning [i.e. reference]; *still*, there must be something on the part of the meaning [i.e. reference] which *corresponds* to this” (Frege 1979e, 119 n. *, emphasis mine). This implies that the unsaturatedness of incomplete senses is primary and that of functions derivative. Thus if incomplete senses are really complete entities with gaps in a merely nominal sense, then functions are too, and even more so, since their incompleteness is derivative.²³

Frege states his metaphors of incompleteness and unsaturatedness for incomplete senses and functions alike. On Dummett’s view, why would Frege even bother with these metaphors in the case of incomplete senses? There would be nothing real for them to be metaphors *of*. On Dummett’s view, perhaps Frege should have said that incomplete senses are *not* like functions because incomplete senses are really complete. Perhaps Frege should have offered metaphors of their real completeness. Perhaps Frege should have said an ostensibly incomplete sense only seems like a cloak, and is really like a body that fills out a cloak, unlike functions, which are really like cloaks. But Frege says the opposite.

Thus Dummett’s way out vitiates Frege’s thesis that expressions, senses, and references stand in a modeling relationship such that all mirror each other (Frege 1971h, 123). Surely it is unrewarding to hold that the mirroring merely implies “there is a so-called incomplete sense,” and not also that it has a gap needing completion. In language, sense, and reference alike, surely Frege intends gaps in whatever is incomplete! If Dummett wants to make a function’s sense into a complex of an objectual sense that is really complete and a merely nominal gap or argument-place that is external to the sense *per se*, this is emendation, not scholarship. If anything, it is the mode of presentation which is the complete constituent the incomplete sense contains. Modes of presentation need no binding because cognition *per se* is nonpropositional. Modes of presentation are the cognitive core of linguistic senses. The context principle governs senses, not modes of presentation. Frege’s containing modes of presentation within senses is his synthesis of cognition with thoughts expressible in language. The extension of this synthesis to entities which can only be grasped in the context of a statement, e.g., to numbers, is Frege’s subtle contribution.

The root of Dummett’s dilemma is his belief that for Frege every entity is either an object or a function (Dummett 1981, 257, 248–54). Dummett is convinced Frege actually says this, citing *Grundgesetze* vol. 1, §2, “I count

as *objects* everything that is not a function" (1981a, 235 n.1). But Frege restricts the paragraph to the "domain of arguments," and Frege never expressly takes senses as arguments. Absent independent evidence that senses are objects or functions, there is nothing here to show that any sense is an object or a function. We do not even need to know what the restriction is, in order to know that not all entities are objects or functions. *Any* restriction would be pointless if all entities were objects or functions. On its face, the restriction is of all *customary* references to objects or functions. In fact, just two paragraphs earlier, which sets the context, Frege distinguishes senses in general from references in general,²⁴ and seems to be mentioning senses only to set them aside until §5. We see the same pattern in "Function and Concept." Frege says, "An object is anything that is not a function," but he starts the paragraph by restricting it to the domain of "arguments and values" (1971b, 32). And three pages earlier, he distinguishes senses from references only to set senses aside for the rest of the paper (1971b, 29). Thus I have never read these texts as saying that all entities are objects or functions, though they obviously seem to say that if they are read out of context.

In *The Interpretation of Frege's Philosophy*, Dummett is still convinced Frege actually says that all entities are objects or functions, citing *Grundgesetze* and "Function and Concept" (Dummett 1981a, 235, 235 n.1). He speaks of "Frege, with his classification of what exists into objects and functions" (1981a, 429). He admits that senses are entities. And he still rejects Peter Geach's thesis that incomplete senses are functions (1981a, 251–53). These three points collectively imply that Dummett still holds that all senses—even incomplete senses—are objects. He says that Frege "distinguish[es] various kinds of object: logical objects; physical objects; ideas and other mental contents; rational beings; thoughts and other senses. . . ." (1981a, 429).²⁵ The rest of the paragraph confirms he means thoughts and *all* other senses (1981a, 429, "Still less. . ."). Dummett (1981a, 393, "But, in general . . .") also confirms this. He duly discusses again in what way senses are incomplete (1981a, 265–70), and seems to take the same way out (1981a, 270).

In light of all this, Dummett's positive statement that incomplete senses are objects is surprisingly weak: "on a non-Fregean use of the word 'concept' [to mean incomplete sense], it *might* be held that 'the concept of citizenship' stood for the sense of the word 'citizen'," where the singular definite article indicates an object (1981a, 90, emphasis mine). The most he is willing to argue is that:

Since the phrases "the sense of the name 'Mont Blanc'," "the thought that the Earth is larger than the Moon," and "Pythagoras's Theorem" do appear to

function as what Frege called “proper names,” there is, for Frege, no question that *at least some* senses are objects.” (1981a, 132, emphasis mine)

He does not mention that “the sense of the incomplete name ‘ξ is a card deck’” functions exactly as much as a proper name as do his examples. This proper name is complete; the “‘ξ’” is not used but only mentioned as part of the indirectly quoted incomplete name.

Montgomery Furth makes complete senses objects and incomplete senses functions (1968, 9). Perhaps the 1993 Dummett agrees, since he says a concept is “a component of a thought” (1993, 61, 105, 129; see my 2003, 65); but perhaps this is his “non-Fregean use” of “concept” again. If Dummett is no longer insisting that all senses are objects, this rescues him from the dilemma of admitting incomplete senses either as complete objects or as incomplete objects. But he still faces the deeper dilemma. If he still insists that for Frege all entities are either objects or functions, his only other options are that incomplete senses are either functions—*pace* his rejection of Geach—or not entities. The 1993 Dummett also says, “All things are *Bedeutungen*” in the sense of being “a possible object of reference” (1995, 8), facing the deeper dilemma.

Dummett criticizes Geach for saying incomplete senses are functions because they map other senses onto thoughts. I agree with Dummett that they are functions only functionally speaking. They do not belong to Frege’s category of functions. But in virtue of their being indirect references, incomplete senses might be called indirect functions. I think we may accept Geach’s innocuous mapping observation, which is not the same as the harmful conclusion he draws from it that incomplete senses are customary functions. If his mapping observation sheds no light on the ultimate nature of senses, *pace* Dummett (1981a, 268), so what? We might also harmlessly note that assertoric force in effect maps thoughts onto assertions. Would anyone conclude that forces are functions, or that this sheds any light on the nature of forces? In this innocuous sense, we can even use objects to map functions onto truth-values! But on Frege’s view that sentence, thought, and truth-condition are structural models of each other, sense-mapping is just what we should expect. The mapping of senses onto thoughts by incomplete senses innocuously models the mapping of references onto truth-conditions by incomplete references.

Dummett criticizes Geach’s view that incomplete senses are unsaturated functions for destroying Frege’s major atoms-and-molecules metaphor of how thoughts are composed of senses (1981a, 263–66). I face the criticism too, since I find incomplete senses unsaturated in their own right. I think Dummett has it backwards. Saturation is a basic *part* of Frege’s atoms-and-molecules metaphor. Frege studied chemistry. And when I described

Frege's functions and concepts to him, the chemist Stephen Richardson immediately identified the basis of the metaphor as chemical saturation. An unsaturate is an atom having electron-spaces available for electrons from another atom to fill, binding the two atoms together into a molecule. Thus the mapping is of electrons onto molecular compounds by unsaturates. Polyadic functions and polyadic incomplete senses are like polyunsaturates. The greater the number of binding completions needed to form the compound, the more unsaturated—and existentially unstable (volatile!)—the compound is. Thus Frege seems to be explicating the metaphysical bond of the unity of a thought as a valency bond. And valency bonds are just what bind atoms into molecular compounds. Now how could saturation, which is the basis of molecular theory, go against molecular theory?²⁶

Dummett argues further that if senses are removed from thoughts to yield incomplete senses, then we must know thoughts first, and cannot learn new thoughts by “atomically” compounding them out of senses (1981a, 267). If so, this is no special problem about senses, since by parity of reason we could not learn new truth-conditions by “atomically” compounding them out of references either. And Dummett still has Frege's saturation metaphor backwards. It is precisely through saturation that we compound chemicals out of atoms. Dummett is confusing the order of cognition with the order of being. In the order of cognition, we can and generally do cognize new thoughts by compounding them out of senses. We can do so precisely because in the order of being, thoughts are parsed in many ways into incomplete senses by removing complete senses.

The truth is that the difference between decomposing thoughts into senses and composing senses into thoughts is as little as the difference between *immortal* and *lives forever* (Frege 1971h, 125). The distinction is only in reason, *pace* Dummett (1981a, 290–91). Frege says:

The mental activities leading to the formulation of a definition may be of two kinds: analytic or synthetic. This is similar to the activities of the chemist, who either analyzes a given substance into its elements or lets elements combine to form a new substance. In both cases, we come to know the composition of a substance. (1971e, 61; see 1979, 202; 1979f, 208–9; 1971d, 42, 43 on chemistry; *Begriffsschrift*, §9)

Indeed, statements *are* definable names for the later Frege, since they are always composed of simpler logical subject- and predicate-names.

Thus Frege's context principle for senses can easily permit and require that subsentential senses be logically derivable as remainders from the thoughts they are parts of, *pace* Dummett (1995, 14). Here we must not be confused by Frege's statement that “the whole reference [of a statement]

and one part of it do not suffice to determine the remainder” (1971i, 65). Frege is right. The True minus the concept *iron* does not tell us which object is said to be iron. Nor would the True minus the Eiffel Tower tell us which concept the Eiffel Tower is said to fall under. But this is a red herring. A *thought* and one part of it *do* suffice to determine the remainder. The thought that the Eiffel Tower is iron minus the sense of “the Eiffel Tower” yields the sense of “() is iron.” Similarly for any sense. This is why definitional composition and decomposition are distinct only in reason for Frege. For definitions can fix references only by fixing senses. Again, for the later Frege, statements are definable names composed of simpler logical subject- and predicate-names.

We must also not be confused by Frege’s statement that “I call anything a proper name if it is a sign for an object” (1971d, 47 n.*). Dummett is wrongly converting this to “If anything is a proper name, then it names an object.” The conversion is restricted to customary references.

Dummett himself cites a text that “an object such as Mont Blanc cannot be part of a thought”(1981a, 533n). Indeed, Dummett knows as well as anyone it is Frege’s famous view that “the object is not . . . itself a constituent of the thought,” and that an object such as a mountain “is not the sort of thing that can be a constituent of a thought” (1981a, 130; see 137–38, 177, 533n). Yet Dummett insists just two pages later that “at least some senses are objects” (1981a, 132). Dummett’s way out is to say that it is only *wirklich* or concrete objects that cannot be senses (1981a, 393). I disagree. Frege’s terminology is misleading. What we would normally call his realism, he calls objectivism. It is objective entities as such that are logically mind-independent, i.e., exist and are as they are independently of the manner in which we think of, regard, conceive, or speak of them. “*Wirklich*” ordinarily translates as “actual” or “real.” Frege’s “*wirklich*” is best translated as “causal.” He has in mind a causal activity or capacity that has nothing to do with the issue of metaphysical realism. Senses are not *wirklich*; they can act on us only in the passive sense of being sometimes grasped by us, and thus affecting how we regard things. That is a *special but secondary* reason why concrete objects cannot be senses. The general and primary reason is that to allow any object, *wirklich* or not, to be a part of a thought would destroy the objectual realism implicit in Frege’s sense-reference distinction. It is not just our accompanying subjective idea, but even our objective sense via which we think about a mountain, that must be distinguished from the mountain itself. Frege says, “When we say ‘the Moon’, we do not intend to speak of our idea of the Moon, *nor are we satisfied with the sense alone, but we presuppose a reference*” (1971i, 61, emphasis mine). And Frege applies this realism, which he calls his objectivism, with the strictest sameness to abstract objects.

If senses were objects, would not Frege have said so? That is a huge category question. What, if anything, does Frege say about it?

Frege says right in “On Sense and Reference” that senses are not objects: “A *truth value* cannot be a part of a thought *any more than*, say, the Sun can, *for it is not a sense but an object.*”²⁷ This is the key text. It is the only text in which Frege says whether senses are objects. Frege says that a truth-value—an abstract object—is not a sense, because it is an *object*. Note the level of generality of this category exclusion. He does not say, “for it is not a sense but a *truth-value*.” He does not say, “for it is not a sense but an *abstract* object.” He does not say, “for it is not a sense but a *customary* object,” as if senses were objects after all—indirect objects. (Senses are, of course, indirect *references*; this is what indirect reference is all about.) He says, “for it is not a sense but an object.” And if no objects are senses, then no senses are objects. The context does not even appear to suggest a qualification. He is saying that the True cannot be a sense, not because it is the True, not because it is an abstract object, and not even because it is a customary reference (on my view there is no other kind of object; functions are the other kind of customary reference), but for the same reason the Sun cannot be a sense: because it is an object. This is close to Aristotle’s principle that the true reason is the most general one; “because it is a customary reference,” which is suggested by other texts which distinguish customary references in general from senses, is more general but is not what Frege says here. Nor is this “smoking gun” text in some obscure writing. It is in *the* paper on sense and reference.

Can we sweep this plain and forthright text under the rug of esoteric argument? The only way out the text allows is to admit some senses as objects which “cannot be a part of a thought.” And that is no way out. All senses must be parts of thoughts, due to the context principle in its application to senses; even thoughts are parts of compound thoughts.

Frege wrote “On Sense and Reference,” in which he says that a truth-value is not a sense because it is an object, and therefore that in general an object is not a sense because it is an object, almost at the same time as “Function and Concept” and *Grundgesetze* vol. 1, in which, Dummett believes, Frege says that every entity is either an object or a function, and “On Concept and Object,” in which, Dummett believes, Frege says that the singular definite article always indicates an object in every possible context, including even contexts which shift reference to indirect reference. Frege never suggests that these four works contradict each other in the least on whether senses are objects or functions. To the contrary, Frege presents the works as being all of one piece. This is easily explained on my interpretation. Looking to the context of each text Dummett cites, Frege is saying only that all *customary references* are either objects or functions, and

only that in contexts of *customary reference*, the singular definite article always indicates an object. But how can Dummett reconcile the texts?

The problem of the sense of expression “A,” which Frege never mentions, is subtler than the problem of the concept *horse*. For there is a tension here between the fact that the singular definite article indicates reference to a complete entity in the case of customary reference, and the fact that directly quoted incomplete names are mentioned not used, so that their incompleteness is not an incompleteness of any expression containing the quoted name. The dilemma is that indirect reference and indirect quotation lie in between customary reference and direct quotation. Or more simply, senses lie in between customary references and names. There are two options. First, if we assimilate indirect reference to customary reference, then “the sense of expression ‘*horse()*’” refers not to a complete *object*, but to a complete *sense* which represents the intended incomplete sense. For indirect quotation is of senses (1971i, 65), and no senses are objects (1971i, 64). Second, if we assimilate indirect quotation of senses to direct quotation of names, then “the sense of expression ‘*horse()*’” can and therefore must refer to the intended *incomplete* sense. For we can and must refer to an intended incomplete *name* if we directly quote it using the singular definite article, since its incompleteness is mentioned not used. The second option is correct. For while incomplete senses are literally and metaphysically incomplete, their incompleteness is mentioned not used in indirect quotation of them. That is, their incompleteness is not a literal, metaphysical incompleteness of or within any sense containing them, if they are being indirectly quoted. For if they are indirectly quoted, they are not literally, metaphysically contained in any accompanying customary sense in the first place. For example, in “The sense of ‘*horse()*’ is an incomplete sense,” the gap in the sense expressed by ‘*horse()*’ is not a gap in the thought the sentence expresses. The thought is complete and has no gap. Thus the second option trumps the first. Ironically, the solution is that due to the mentioning nature of indirect quotation, there *is* no problem of the sense of expression “A” like the problem of the concept *horse*. And perhaps that explains why Frege never mentions the problem.²⁸

Twelve years later Frege repeats virtually the same text in a letter to Russell: “*Truth* [i.e. the True] is not a component part of a thought, *just as* Mont Blanc . . . is not . . .” (1980, 163, emphasis mine). The text is so much the same that surely we must impute the “for it is not a sense but an object.”

As Frege explains in a third text in another letter to Russell:

Now a class cannot be the sense of a sign, but only its meaning [i.e. reference], as Sirius can only be the meaning of a sign, but not its sense. Hence a class

cannot be the indirect meaning of a sign, *any more than* Sirius can. . . . Can any class whatever be a component part of a thought? *No more than* the planet Jupiter can. A class . . . can be defined in different ways. . . . If the class was part of the thought that an object p belonged to it, then the change in the sense of the class name would not affect the thought, provided that the class itself remained unchanged. . . .

Or does a proposition have a meaning [i.e. reference], and this is a thought? If the latter, then the propositions ' $2^3 > 7$ ' and ' $3^2 - 1 > 7$ ' would have to designate the same thought [for they have the same reference]. Now the thoughts contained in those propositions are evidently different. . . . We are thus compelled to regard a thought as the sense of a proposition. (1980a, 157, emphasis mine; see 158)

Note that logical equivalence is not the test of identity of thoughts. The natural reading is that since objects are customary references, and customary references remain the same across changes in sense, no objects are senses. That is, the reason why objects are not senses is that objects are extensional in sense (2) and senses are intensional in sense (2). Frege is telling us that we do not understand his solution to his puzzle about informative identity statements unless we understand that no senses are objects or functions.

Frege repeats this in a fourth text in “[Notes for Ludwig Darmstaedter]”:

The object and concept are not constituents of the thought expressed. . . . A distinction has to be drawn between the sense and meaning [i.e. reference] of a sign. . . . If an astronomer makes a statement about the moon, the moon itself is not part of the thought expressed. The moon itself *is the meaning* of the expression ‘the moon’. Therefore *in addition to its meaning* this expression *must also* have a *sense*, which *can* be a constituent of a thought. . . . This is what makes it *possible* for a sentence of the form ‘ $A = B$ ’ to express [an informative identity]. (1979g, 254–55, emphasis mine)

The natural reading is that there must be senses *in addition to* objects and concepts. For Frege’s expression “object and concept” indicates that he makes the sense-reference distinction generally for all customary references. Some functions, such as *the capital of* ξ , are not concepts, but I see no reason for them to be an exception to the general distinction.

In a fifth text in “Introduction to Logic,” Frege asks very generally, “Now what has the object got to do with the thought?” Mont Blanc is the example, but his argument applies to abstract objects as well, since there can be thoughts about the axis of a fictitious planet or about the odd and

even integer, even though such abstract objects do not, and in the latter case cannot, exist. Frege then says that “the same point can be approached” by explaining in general how informative identity statements are possible (1979d, 191–92; see 187).

Frege adds in a sixth text that numbers cannot be parts of thoughts: “while the sense of a number sign can be part of a thought, a number itself cannot” (1980a, 158; see 163 for a seventh text on the number 7).

Thus Frege denies that three kinds of abstract objects—truth-values, classes, and numbers (numbers are classes of classes for the later Frege)—are senses. If that were all, Dummett’s view that senses are abstract objects would emerge as Ptolemaic, with truth-values, classes, and numbers as big epicycles. Dummett could still say that senses are a special class of abstract objects, namely, objects that are indirect references. He might say that in all the works in which to all appearances Frege is introducing senses as a unique category to do a job objects cannot do, Frege is really distinguishing objects which are not senses from other objects which are, but without ever telling us his secret meaning. He might even say that while a *truth-value* is not a sense but an object, a *sense* is both a sense and an object. But all this is curiouser and curiouser. Frege’s plain meaning is that senses and objects are different categories. A sense is not an object but a sense. An object is not a sense but an object. And Frege is most naturally read in all these texts as arguing for the even more general thesis that *no* customary references are senses.

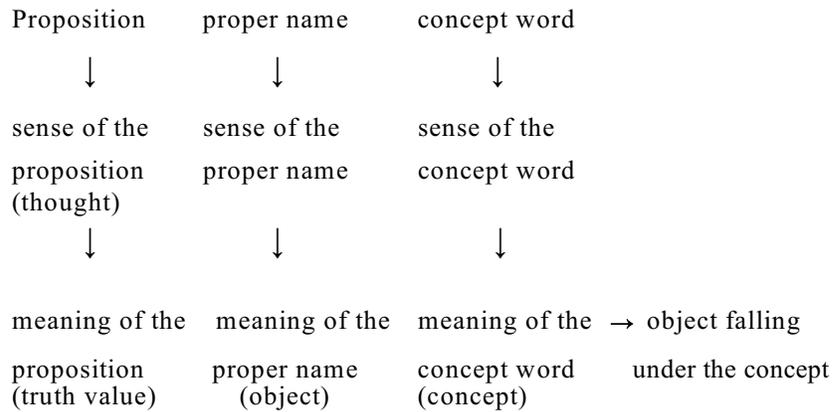
Frege rarely argues that concrete objects in particular cannot be senses. He says “one can see Etna, but one cannot see the thought that Etna is higher than Vesuvius” (1979f, 225). He says “the sense of the word ‘sun’ is not somewhere in space, nor does it have mass” (1980e, 128). We can see why he has little interest in giving such arguments: they are shallow and overkill.

“On Sense and Reference” and “[Comments on Sense and Meaning]” read so much more easily when we think of senses as a unique category that we can see how badly Dummett is bewitched by his belief that Frege says every entity is a function or an object. For example, Frege explains objects as the customary references of proper names. Frege says, discussing only the customary reference of a proper name, “I call such a meaning [i.e., customary reference] an object” (1979c, 119). Is that the role of any senses? Quite the opposite. Senses are introduced precisely because the category of objects is inadequate to explain the possibility of informative identity statements. Frege says “it is *via* a sense, and only *via* a sense that a proper name is related to an object” (1979c, 124); the word “object” is used without qualification. This implies that if senses are objects, they are named only *via* senses. If, then, senses *are* objects, is there not a vicious regress of

objects? Or is the category of objects then adequate to explain the possibility of informative identity statements after all?

This is not to mention the diagram in Frege's letter to Husserl with all the senses on one line and all the references on another. Frege says:

The following schema should make my view clear:



(Frege 1980d, 63; brackets omitted)

Frege places objects on only one line, the line of references.²⁹ Dummett is placing objects on both lines. If senses are objects, why does Frege not write "object" for the three senses on the line of senses? Or if only complete senses are objects, why does Frege not write "object" for the two complete senses? The answer is in "On Sense and Reference": for they are not objects but senses.³⁰

This pulls the realm of senses out of the realm of customary references, *pace* Dummett (1981, 680). In fact, Frege's distinction between senses in general and references in general already does that. This answers the question, Just how tightly does Frege tie linguistic function to ontological category (Dummett 1981a, 384)? Namely, tightly enough to pull the whole category of senses out of the category of customary references. And by parity of reason, tightly enough to pull tones and forces out of that realm too (Frege never calls them objects or functions). For if anything, they function even less like customary references than senses do. Ideas never were in the realm of customary references. Frege's private language arguments require that *all* references, customary and indirect alike, be objective, that is, mind-independent. (As indirect references, senses do belong to the realm of *all* references. Thus when Frege distinguishes senses from references, he can only be distinguishing them from the realm of customary, i.e., direct, references.)

How can we refer to the tone T or the force F , if they are not objects? Assimilating reference to tones or forces to direct quotation of names or other expressions seems too implausible to consider. But there are three other models of reference in Frege: reference to functions, to ideas, and to senses.

First, “the function F ” refers to an object, a course-of-values, which technically represents the function. We might say that in *saying* something about the object, we *show* something about the function it represents. Can objects represent tones or forces?

Second, we can “take” an idea “as” an object. We might say that in *saying* something about the ostensible object, we *show* something about the idea. Can we “take” a tone or force “as” an object?

Third, we talk about the sense of expression “ A ” as an indirect reference in indirect quotation. Here we say an indirect sense, showing the indirect reference. Indirect references are presented only via indirect senses. Is reference to tones or forces part of indirect speech?

All three models anticipate the early Wittgenstein on saying and showing; I think the first model might have influenced him.

I think the first model is inappropriate. Granted, tones and forces need something to apply to. But if Frege thought they were incomplete, then his notation would have argument-places for their arguments. That is, if they were incomplete, I think Frege would have said or at least shown so. Of course, there is the innocuous sense in which items in any two or more related categories can map each other onto something. Sad tones can map utterances onto sad utterances, and assertive force can map utterances onto assertions. But in that innocuous sense, truth-values can map functions onto objects, reversing the metaphysical order of completeness, and reversing arguments and values as well. Of course, on an ontologically serious mapping reversal, objects and functions must be “converted . . . or, speaking more precisely, represented” as appropriate (Frege 1971d, 46).

The other two models may seem inappropriate as well. Tones and forces are neither wholly objective nor wholly subjective on my view, and they are not ways to identify anything. But I shall argue that the second model is workable, and that the third model is best.

Are ideas, tones, and forces objects because “the idea my assertion of ‘ A ’ produces in you,” “the tone of assertion ‘ A ,’” and “the assertoric force of assertion ‘ A ’” must always refer to objects? The quick answer is no, since objects are essentially and wholly objective.

As we saw, Dummett lists ideas as a subcategory of objects (1981a, 429). That would be a quick way to rescue idealist intuitionism from psychologism (Dummett 1981, 684)! Indeed, it would rescue psychologism from psychologism. For it makes nonsense of Frege’s twelve private

language arguments (my 2003, 289 n.1) and his cautious verdict that ideas can at best be “taken as” objects (1971i, 60; see 1968a, 510). The verdict implies that ideas are *not* objects. Why would Frege suggest that ideas can be “taken as” objects, if Dummett is right that ideas already *are* objects? On Dummett’s account, should not Frege be suggesting that ideas—or *private* objects—can be taken as *public* objects? But Frege speaks of objects without qualification. The verdict also implies that the arguments aim to show not just that our thoughts are objective, but that the things we talk about, i.e., our references, are objective. This double aim merges in indirect reference. Even in *Grundlagen*, ideas are not objects. Frege makes ideas *subjective Vorstellungen* and objects *objective Vorstellungen*; these are mutually exclusive categories (§27, n.1), *pace* Dummett (1991, 225). Why would Frege place ideas and objects in mutually exclusive categories, if Dummett is right that ideas are objects? Frege is saying that objects are objective and ideas are subjective. Splitting these *objective Vorstellungen (Inhalten)* into senses and references, the later Frege’s objects are just as totally excluded from being ideas. The later Frege says to Husserl, “objects and concepts have the same objectivity” (1980d, 63). Do ideas have this same objectivity too?

This pulls “the realm of the purely mental” out of the realm of reference, *pace* Dummett (1981, 680). This completes our separation of three realms (references, senses, the mental) Frege repeatedly distinguishes (1971i, 60–61; 1968a, 523–24), but Dummett commingles.³¹

Making tones into objects would spare Dummett the difficulty of having to argue that tones can be objective (1981, 84–89). For they would all be objective as a matter of course. But Frege says, “Such colouring and shading are not objective,” and insists they are really ideas belonging to art (1971i, 61; 1979e, 145). If so, tones cannot be objects, functions, or senses. But in other texts he discusses the objective aspects of language which give rise to the ideas, such as choice of word, tone of voice, and onomatopoeia (1979e, 127–28; 140; 1968a, 514–15), and even seems to call tones “constituents of sentences” (1968a, 514). I think the best view is that tones necessarily involve both objective aspects of language use (or, more generally, of behavior), and subjective ideas the aspects cause in us. A very plausible gloss would be that tones are really groups (extensions of classes) of (normally resembling) subjective ideas (across persons), namely groups of moods or feelings caused by certain objective language uses. This is plausible because Frege seems to say tones are really ideas. That would place tones on the subjective side of the house. But if so, my criticism of Frege is that this only sweeps the emergent nature of tones under the rug. Taken by themselves, neither the objective language behaviors nor the subjective ideas they cause are (successful) conveyances or expressions of

mood or feeling. Conveyance or expression is essentially relational and clearly an emergent property. Thus tones are or ought to be emergent entities with objective and subjective components. If so, they cannot be objects, functions or senses, insofar as objects, functions, and senses are wholly objective; and they cannot be ideas, insofar as ideas are wholly subjective. This may seem to violate Frege's principle always to separate the objective from the subjective. But the components are separate. And if there can be no relations among entities in different categories, we will all be in trouble, except for kind-monists. This is not to mention how tightly Frege ties linguistic function to ontological category. Thus the best gloss is that tones are emergent entities for Frege. And if tones are special kinds of emergent entities, this would provide an excellent explanation of why they are a unique category. It would also remove the mystery about their nature. For the concept of an emergent entity is a merely logical concept. An emergent entity is merely an entity that has at least one property none of its logical constituents has.

Similarly for forces. Frege says assertion is the *objective* communication of a judgment, where a judgment is an indefinable sort of *inward* recognition (1979e, 139; 1968a, 513; see 1971i, 65; 1971h, 126, 126 n.*). A very plausible gloss would be that forces are really kinds of objective speech acts, namely acts which communicate inward states such as recognition or query. This is plausible because the assertion sign (judgment stroke) is part of the objectively communicable formal notation. That would place forces on the objective side of the house. But if so, my criticism of Frege is that this only sweeps the emergent nature of forces under the rug. Taken by themselves, neither the objective nor the subjective components of assertions are outward communications of inward recognitions. Such communication is essentially relational and clearly an emergent property. Thus the best gloss is that forces are emergent entities for Frege. But we already know assertion cannot be an object or a function. For Frege says the assertion sign (judgment stroke) cannot function as a name, on pain of reducing assertions to suppositions (*Grundgesetze* vol. 1, §5; see 1968a, 514); and we have seen how tightly he ties linguistic function to ontological category.

Thus tones and forces are unique categories; they are irreducible to each other. But we can "take" them "as" objects by first "taking" their subjective components "as" objects, and thus refer to them as easily as we can refer to ideas. I call this a "double take" (pun intended). Thus the second model seems workable for tones and forces, if Frege's single take works.³²

The third model is best. Tones, forces, and explications are part of speech as much as senses are. They have no use outside of speech, and their use is what they are. Even a happy humming or a commanding look is

expressive in some sense. Thus we should, after taking their subjective components as objects (a single take), construe reference to the tone T, the force F, or the explication E as indirect reference by analogy to indirect reference to the sense S. In contrast, reference to objects we double take such entities to be on the second model is direct reference.

To sum up, Frege provides reasons why senses, ideas, tones, and forces cannot function as—and so cannot be—objects or functions. Yet Dummett reduces Frege to two categories, objects and functions. Because he honestly believes Frege actually says this, he is constrained into pounding square and triangular pegs into round holes, and reducing Frege's jungle to a desert.

Dummett is steering a heroic course in the face of great difficulties, pursuing a mirage of Frege as an austere dualist with two categories, objects and functions. Frege's austerity lies deeper, in his severe tie of ontological category to linguistic function. This results in a profusion of categories due to the profusion of linguistic functions. The textual difficulties vanish. We also see the whole linguistic turn. Dummett is right to pinpoint the turn in Frege's requirement that for objects to be named, a criterion for their identity must be provided. But objects are just the tip of the iceberg.

6. THE PROBLEM OF BIFURCATION

A chief problem for Dummett is how to keep Frege's later context principle from bifurcating into one principle for senses and another for references (Dummett 1995, 10, 14). Dummett links them backwardly in the cognitive order by deriving senses from references.

I double-link them. Senses are logically tied to references by being ways of presenting references. This links them on the forward road in the cognitive order, and on the backward road in the ontological order. Due to the ontological dependence of modes on objects and functions, senses and references are not wholly distinct from each other as categories, even though there may not always be a reference. For surely a reference is not wholly distinct from the ways in which it can be given. Not every distinction in reason involves symmetric dependencies. This is not to mention that on the descriptivist theory of senses, every definite description expresses a sense which is essentially descriptive of an extensional in sense (2) concept under or within which its reference, if any, falls.³³

The leap from sense to reference is especially transparent for primitive logical entities such as the identity relation. Here the leap is between not wholly distinct entities in the strong sense that primitive references cannot fail to exist if the senses reflecting their nature do, entailing symmetric ontological dependencies. This is the later Frege's implicit atomistic version

of the Parmenidean thesis that the rational is the real and the real is the rational, though Frege has a more general and somewhat different version in his thesis that every true or false sentence must have a referring logical subject (1971i, 62). It is in any case the later Frege's implicit version of his own earlier thesis that even some complex logical entities, namely numbers, are transparent to the reason (*Grundlagen*, §105).³⁴

7. THE CONSEQUENCES FOR INTUITIONISM

By parity of reason, there is no backward road from truth to propositional sense.³⁵ This is fatal to intuitionism, and to strong verificationism. It does not matter whether a propositional sense describes a truth-condition, or whether it is logically tied to a truth-condition at all. What matters is that the truth-value, truth, is propositionally presented in indefinitely many ways in the order of cognition. Thus there is no backward road from truth to propositional sense in the order of cognition, any more than there is from truth-conditions. We cannot squeeze an intension in sense (2) out of all the extensions in sense (2) in the world, even including *assignments* of truth-values among the extensions. My point is so general, it does not even matter if the assignments of truth-values are based on proofs or not. We may say that the truth of a statement underdetermines the thought it expresses.

A second problem of intensionality in sense (2) arises more specifically for intuitionism in that there can be many different proofs of the same thesis, so that there is no backward road from the truth of a thesis to any one proof as the sense the thesis expresses. As Frege says, "Frequently several routes for a proof are open" (*Grundgesetze*, p. 3; see Dummett 1981, 634). Thus we may say the truth of a thesis underdetermines its proof.

We cannot need to be guaranteed we will discover an object in order to understand the sense its name expresses, since we cannot need to single out that object in order to understand that sense. For an objectual inquiry to be genuine, it must be at least zetetically, i.e., investigatively, possible for the object to turn out to have properties other than those we think it has, or not to exist at all. By parity of reason, we cannot need to know how to prove that a statement is true in order to understand its sense, because we cannot need to have the truth in order to understand its sense. For the inquiry to be genuine, it must be at least zetetically possible for the statement to turn out to be false. These two points coalesce for Frege, for whom truth *is* an object. I need not add that the order of investigation is a form of the order of cognition.

This is the problem of Plato's *Meno*. How can it make sense to inquire

about anything if the object of inquiry must already visibly contain the answer? What would we be discovering?

The problem of how informative inquiry is possible is more generally the problem of how informative identity statements are possible. Thus the *Meno* problem is more generally Frege's puzzle. And for Frege, for a statement "to be informative—to have 'cognitive value'—it is necessary that a mere knowledge of its sense be insufficient to guarantee a recognition of its truth" (Dummett 1981, 228). The burden is on the intuitionist to explain why the *Meno* problem and Frege's puzzle are not genuine puzzles.

Can an intuitionist find any sense of discovery in moving from having an effective means for obtaining a proof ("informal proof" or "demonstration") to obtaining a proof ("canonical proof") (Dummett 2000, 270–71)? No, this is illegitimate on intuitionism's own showing. For the expressions "an effective means" and "a proof" are existential generalizations. And according to intuitionism, the only way to prove an existential generalization is to prove that there is an instance.³⁶ Thus the only way to prove "*There is a proof*," "*There is an effective means to a proof*," or even "*This is an effective means to some proof*," so that these assertions have meaning, is to produce an actual proof. Thus there can be no sense of informative inquiry for intuitionists, since for them a statement can have no meaning until we have already proved (or disproved) it. Thus intuitionists cannot discover whether intuitionism is true or false. This is not a problem of recognition or of circularity, but an application of the intuitionist rule for proving an existential generalization. If there is no actual, specific proof or disproof of intuitionism, then intuitionism is neither true nor false, on intuitionism's own showing.

Dummett says, "From an intuitionistic standpoint . . . , an understanding of a mathematical statement consists in the capacity to recognize a proof of it when presented with one; and the truth of such a statement can consist only in the existence of such a proof" (Dummett 2000, 4; 1978, 8). But "a proof" is an existential generalization, and so is "the existence of such a proof"; and the intuitionist always requires an instance. Thus for the intuitionist, a mathematical statement has neither sense nor truth-value until an actual, specific proof or disproof is found. Nor has any actual, specific proof been given of the general intuitionistic thesis just quoted. What proof could there be? Certainly not a mathematical one!

Dummett defines intuitionist $\neg A$ as its being provable that there is no proof of A (Dummett 2000, 11). But if I have just proved that there is no proof of intuitionism, then $\neg(\text{Intuitionism})$.

If the general intuitionistic thesis needs no proof to be meaningful, why do mathematical statements need proofs to be meaningful? Both involve

generalizations; and if anything, the general thesis is more general.

Can intuitionists informatively move from nonintuitionistic meaning to intuitionistic meaning? If so, then our interest in intuitionism is destroyed. For then mathematics has nonintuitionistic meaning after all (see Dummett 2000, 250–51; ix).

Can intuitionists informatively move from mere strings of symbols to intuitionistic meaning? If so, then intuitionism is a mere appendage or post-script to formalism. And if proof-strings have formalist meaning-in-use, then mathematics has nonintuitionistic meaning after all.

It seems, then, that intuitionism cannot explain away our sense of mathematical discovery even as a mere appearance. I mean that if what is proved is meaningless until it is proved, we cannot meaningfully *look for* anything. Phenomenologically, the intentionality is gone. Is not human creation, such as intuitionists take mathematics to be, normally a teleological process with an end in mind? Is not trying to prove a thesis a paradigm of having an end in mind?³⁷

If different proofs provide or constitute different meanings for a theorem (see Dummett 1981, 674–75), the question arises whether they prove the same theorem, or whether the theorem is ambiguous. But this question is meaningless for the intuitionist unless there is an actual, specific second-order proof of sameness of the theorem. But if there are different second-order proofs of sameness of the theorem and so on, then there is a vicious regress involved in proving whether different provers are ever proving the same theorem.

Perhaps Dummett's belief in intuitionism has influenced his interpretation of Frege, or vice versa. Perhaps his own contextualism and his forced choice argument have influenced both. Dummett makes it clear that Frege is no intuitionist (1981, 586–87, 589).³⁸ But intuitionism and Dummett's program have something essential in common. Namely, both take the backward road. Intuitionism holds that only a proof of the truth or falsehood of a mathematical theorem can provide a thought for the theorem to express. Dummett's program holds that for Frege, the truth-condition of a statement wholly determines which thought the statement expresses.

In any case, Dummett takes the backward road in all three traditional areas of logic: terms (senses via references), statements (propositional senses via truth-conditions), and inferences (theorem senses via proofs of truth), in his Frege scholarship and/or his philosophy.³⁹

It may not seem so, but I agree with most of Dummett's views on Frege. Whatever merit this paper has is largely due to Dummett. For over a quarter of a century, I have regarded him as my Frege teacher from afar. I owe more to him for my understanding of Frege than to anyone except Frege himself. He will always be the world's best Frege scholar to me. I

want to thank him for over fifty years of wonderful service not only to the community of Frege scholars, but to the world of philosophy at large. Thanks to his devotion and integrity, and his ability to bring out the huge volumes of thought implicit in Frege, he has done more than anyone to bring Frege to his rightful place in philosophy.

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NOTES

1. Dummett prefers the term “referents” (1981, 94; 1981a, 2).

2. See my 1979 (59–85), summarized in my 1982 (2–5), and updated in my 1996 (104–9) / 2003 (104–9). All three provide a specific analysis explaining why Frege’s formally explicit definition of Number is implicitly and functionally a contextual definition, in contrast to Dummett’s general and conclusory paragraph (1995, 4–5). The heart of the analysis is that once we see that parallel completions of (A) identity statements about numbers, (B) statements of the equinumerosity of certain concepts, and (C) identity statements about the extensions of those concepts, are logically equivalent, the mediation of (B) can drop out and the identification of numbers with the extensions can take on a life of its own (and this takes us to the later Frege). The end result is an explicit definition, just as Dummett says, but this is just a husk covering up the contextual work of (B)’s mediation. I mailed a copy of my 1979 to Dummett in 1979. He never replied. I wonder if he ever received it. Imagine if he had recanted in 1979!

3. Strictly, about the object that represents identity, namely, a double course-of-values.

4. The geometry permutation is also a private language argument (compare Dummett 1981, 638–42). In *Grundlagen*, Frege uses private language arguments to show that numbers are mind-independently objective long before he uses the context principle to define numbers.

5. This is not Quine’s sense, which involves a fact of the matter (Gibson 1987, 147; Quine 1987, 155).

6. *Grundgesetze* vol. 1, §§0, 30; Frege 1980c, 36–37; 1971d, 42–43; 1971i, 59–61.

7. Frege 1979j, 271; see 1980c, 37; 1979f, 209; 1971e, 53, 59; Kluge 1971, xxvii.

8. Thus the Heijenoort-Hintikka critique of Frege’s distinction between metalanguage and object language is misplaced. For Frege, the distinction has nothing to do with whether his quantifiers range over names as well as objects. For him, the key issue is that science must begin with explications, on pain of vicious regress of definitions. Frege demands that explications occur in a propaedeutic because they are too uncertain to belong to rigorous science. This is what requires and justifies his use of metalanguage. Even if Frege’s quantifiers range over names

as well as objects, he can still make the distinction by using predicates such as “ x is a name of y ” and “ z explicates name x .” The quantifiers would then be the vehicles for general statements about explication (my 2003, 82; compare Dummett 1981, 226). Indeed, what better way is there to talk about general relationships among names and objects, in object languages *or* metalanguages? In Neurath’s metaphor, formal object languages and formal metalanguages are rebuilt ships at sea, built from the timbers of a natural language. Thus to call an object language level 0 and its metalanguages levels 1 . . . n is to go in the wrong direction. It is the natural language which is the level 0 starting point, and the object language is the level n result. Any formalization of a natural language ought to preserve its capacity for referring to object languages and metalanguages alike, as well as to their relationships. And “Carnap . . . has . . . shown that a language can without self-contradiction be used in the analysis of itself” (Ayer 1952, 71; see Carnap 1959, xiv, 53). That is, a language can function as, and so be, both an object language and a metalanguage. Within a universal logic, it is not only possible but necessary to discuss itself as an object language metalinguistically. Of course, we can *also* discuss a universal logic from without, i.e., from within another logic.

What reason is there to suppose that Frege’s quantifiers do range over both names and objects? Frege seems to use German as a metalanguage to explicate the semantics of his eight primitive terms informally as object language, then to formalize their use through regimentation; and all his defined terms are mere abbreviations. He knows the use-mention distinction, and his formal notation seems only to *use* names. But I think he may admit three systematic shifts of his referential use of names. First, I argue elsewhere that for Frege, identity statements assert that two names refer to the same reference (my 2003, 42–65; 1981, 31–36; 1979, 34–50). If so, Frege’s identity statements are metalinguistic, yet occur within the formal notation; he has a formal identity sign. My view implies that Frege quantifies over names as objects in $(\exists x)x = a$. In my 2003 (54), I see this as a formal oddity, a systematic shift of reference in identity contexts, and technically harmless for doing proofs. Now, the identity sign is one of the eight primitive terms. That is because definitions are identity stipulations that two names express the same sense and refer to the same reference, so that definition always presupposes identity (Frege 1971c, 80–81). And Frege has a formal definition sign as well. Second, then, all definitions in the formal notation are metalinguistic. Thus on my view, both asserted identities and stipulated identities are metalinguistic, yet within the formal notation. Third, Frege says “there are two wholly different cases [where] we speak of existence. . . . In the one case the question is whether a proper name designates . . . ; in the other, whether a concept takes objects under itself” (Frege 1971, 104). Surely it would be odd if Frege could not state his own first kind of legitimate existence-talk in his own notation by a systematic reference shift, or simply by introducing names of names and a name for designation.

Frege’s quantifiers do not presuppose names or objects. Frege is very clear on which eight logical names are primitive. They include the universal quantifier and negation, and he defines the existential quantifier directly in terms of them. One must not be confused by the regimentations. They are not definitions. He provides them precisely for his primitive names, which already have explications.

Is there any circularity in the informal *explications* of Frege's primitive logical names? Due to the logically contingent nature of explication, the answer must be no, even if some terms which are or later become defined terms are used or presupposed in the explication of a primitive term. Thus I am not even bothering to check for that. As Frege would say, we must begrudge him a pinch of salt. As Whitehead and Russell say, "a preliminary explanation of the notation [can] place lucidity before correctness" (1950, 1). Indeed, after distinguishing the order of discovery (or of pedagogy) from the order of logic (Russell 1985, 16), Russell adds that for primitive terms, "it must be strictly impossible to *say* what they mean, except by a tautology, for it is with them that language begins" (1985, 26). And Russell is not even discussing contingent explications such as Frege's.

Since there is no circularity in Frege's series of definitions either, circularity is not the cause of Russell's paradox in Frege. The assumption that every concept determines a class is an assumption (*Grundlagen*, §§69 n. 1, 107), and Basic Law 5 is an axiom. The assumption and the axiom may be false, but they are not circular. Whitehead and Russell say the assumption "is perhaps the most fundamental case, of the vicious-circle principle" (1950, 39). This is strictly incorrect. Certain viciously circular *terms*, such as "class of classes not members of themselves," appear to *falsify* the assumption. But many innocent terms have "self-reference or reflexiveness" (1950, 61), and "generate a totality containing members defined in terms of itself" (1950, 64). Thus circularity in this sense is not a sufficient condition of viciousness, and it remains to explain the viciousness of any viciously circular terms. To say the problem is viciousness is merely to pose it. There is at present no definition of viciousness which is neither too broad nor too narrow, or even a specific explanation of Russell's paradox which is any deeper than the mere fact of the paradox (compare Jean van Heijenoort).

I turn from diagnosis to prevention. Self-referential circularity is a necessary condition of Russell's paradox. Ramified type theory eliminates the paradox, and much else, by eliminating all such circularity. Is then such circularity the cause of Frege's *failure to prevent* the paradox? Dummett says, "The stipulations governing the primitive functors . . . could be determinate only if the domain, consisting wholly or largely of value-ranges, was determinate; but the domain was in process of being determined by fixing the *Bedeutungen* of the value-range terms, and so the procedure went round in a circle" (1995, 18; see Dummett 1991, 215, 233, 239, 318; 1981, 529–41). "That is to say, a function is not a well-defined function unless all its values are already well-defined" (Whitehead 1950, 39). But regimentation does not involve such circularity. Regimentation starts with the primitive terms, and thereby all terms, already explicated as expressing fairly clear senses. Regimentation merely aims to make these senses logically determinate, so as to ensure that all terms have logically determinate references. Now, without knowing the cause of the paradox, the only way to regiment without overkill or underkill may seem to be not principled or general, but individually for each sentence in which any paradoxical term occurs, an infinite task not in our power to complete. But in theory, Russell's paradox can be prevented by regimenting each sense so as to impose a determinate truth-condition on each sentence, thereby ensuring a reference for each term. There would be *mutual determination* of all references by

their being regimented in terms of each other, but no circularity, due to the explications. Regimenting is like using scissors to make rough pieces fit in a jigsaw puzzle, or using a saw to make rough planks fit in Neurath's ship, or training civilians into a regiment of soldiers. Any paradoxical sensial guidance from a sense would be individually regimented, and any innocent sensial guidance would remain. Or second, we could generally stipulate that every sense meta-determine that if it determines a paradoxical class, then that class is a member of itself. Or third, we could generally stipulate that every paradoxical sense is barred from the notation, and thereby every paradoxical function along with its paradoxical totality of values. In any of these three ways, every concept named in the notation will determine a class. Frege's paradox is that he should have shown more faith in his commitment to classes as abstract entities. This commitment is established in the *Grundgesetze* preface by his private language argument, long before he tries to prove in vol. 1, §29 that his names refer. The proof is a mere question of slicing up the objective realm into determinate entities; and Frege permits our slicing it as arbitrarily as we please. Thus Frege need not provide any principled solution to Russell's paradox in order to preserve logicism. See my (2003, 100), *pace* Dummett (1991, 211). Of course, my second and third suggestions are principled in preventive ways. Note that explication, used as sensial guidance on how to regiment senses and thereby references *truth-functionally*, is extensional in sense (1). Frege regiments extensional in sense (1) references via extensional in sense (1) senses. Senses and references alike are extensional in sense (1); senses are intensional in sense (2).

The Heijenoort-Hintikka critique seems based on the assumption that for Frege, names *are* objects. And that might be based on the assumption that for Frege, every entity is an object or a function. Now, Frege's names are very plausibly glossed as signs (physical objects) expressing immaterial senses. Thus it might seem that they are not wholly objects, since their component senses are not objects. And it might seem to follow that if we can directly refer to them as signs expressing senses, we can also indirectly refer to them as senses expressed by signs, and that either way, they are not strictly objects, so that strictly objectual quantifiers should not range over them. But Frege makes it clear that all signs are objects (1971, 194). Even the equality (identity) sign is an object (1971, 194). Thus names cannot be a class of signs, namely signs expressing senses. For all signs are objects, and all objects are complete; but many names are incomplete, including the name for equality. (Their incompleteness can only be due to their sense.) Thus names are not objects. The charitable gloss is that names are emergent entities. Their emergent property is that of explaining the possibility of an informative identity statement. Neither mere signs (nor names considered as mere labels) nor mere senses have that property. Making names objects by making them a class of signs would sweep their emergent nature under the rug.

9. As Dummett says, most of the objects Frege names in his formal notation are value-ranges (1995, 18). But Frege is talking about an ideal language for science in general—a *lingua characteristic*—not a *calculus ratiocinator* (*Begriffsschrift*, 6–7; see 2–3). And he uses statements such as “All men are mortal” to explicate quantification.

10. See Haaparanta 1985, 38–40, 46 n.26; my 1999, 116; my 2001, 26.

11. Dummett says Frege rejects facts as a category (Dummett 1981a, 176–77) until 1918, when Frege assays facts as true thoughts in “The Thought” (Frege 1968a, 531; Dummett 1981, 659, 662). If so, this makes facts emergent entities with a thought and an object—the True—as components. There is nothing like this in Russell or Wittgenstein. But it might be better to say that Frege is merely analyzing an ordinary sense of the word “fact.”

12. The following four items in Frege must not be confused with each other: (a) the abstract (noncausal) extensional in sense (2) concept *red*; (b) the abstract intensional in sense (2) sense expressed by “red”; (c) a concrete (causal) extensional in sense (2) red object in the physical world; and (d) an extensional in sense (2) mental sensation of red, normally had by one when one perceives a red object. Now, if (d) is concrete (causal), or even has normal or lawlike patterns of occurrence, then to that extent (d) is objective, *pace* Frege’s classification of ideas as subjective and in flux, and *pace* his sharp separation of the objective from the subjective. Frege allows, and I think requires, at least resemblances across different persons’ ideas even to explain how art is possible (1971i, 61).

13. Thus Frege would reject Dummett’s solution of the *oratio obliqua* problem. Dummett is very clear that his solution is “emendation,” not scholarship (1981, 267, 268; 1981a, 87).

14. Dummett says Frege found that the traditional concept of abstraction is irrelevant to the notion of sense, and does not use it in his final account; but Dummett admits that this final account concerns the formal notation and not ordinary language (Dummett 1981, 676–78). This is consistent with my view that Frege never retracts his early paper (1972) on the basic role of abstraction in learning ordinary language.

Frege denies that traditional abstraction can yield a logically adequate concept of number (*Grundlagen*, §§34, 44; 1971c, 84–85). But in *Grundlagen*, §89, he does not repudiate altogether Kant’s principle that concepts without percepts are blind, which is the heart of abstraction, but only for numbers. Indeed, he calls this “the mistake of supposing that a concept can *only* be acquired by direct abstraction from a number of objects” (§49, emphasis mine). This implies he thinks some concepts *are* acquired by abstraction. Late in life, he indicates that we use abstraction to teach and learn our first “kindergarten numbers” in ordinary language (1979h, 276; see 1979a, 280); perhaps he thinks such teaching is a type of explication (1979j, 271). Perhaps the best view is that from 1884 on, he thinks that not every concept can be acquired by abstraction, but that some must be. This is parallel to Dummett’s distinction between primary and secondary objects. See my 2003 (240–42).

15. One might object that modes in general are ontologically dependent, and that thus perhaps modes of presentation contained by incomplete senses are incomplete too. If so, the chief difference between such senses and the modes of presentation they contain is simply that the former have a linguistic role and the latter are their cognitive components. But surely the modes of presentation contained by complete senses are just as dependent, yet are complete. Thus I see no need for incomplete senses to contain incomplete modes of presentation.

16. My interpretation is in my 2003 (275–77).

17. The conclusion is clear enough to show that Russell interprets Frege the

same way I do. For Russell is criticizing Frege precisely for *not* holding that every denoting phrase denotes a denotation (reference) as opposed to a meaning (sense). That is, Russell is agreeing with me that Frege holds that some denoting phrases denote meanings (senses) *as opposed to* denotations (references).

18. Dummett is well aware that difference in reference implies difference in sense.

19. Dummett is right that references are singled out only via senses; Frege expressly says so (1979c, 124). But I criticize Frege in the following limited sense. If dogs can single out the Moon without the use of imperceptible concepts in *Grundlagen*, §31, then dogs can single out the Moon without the use of imperceptible senses in *Grundgesetze*. Of course, the Moon is not a linguistic reference for dogs, but merely a physical phenomenon for them (see my 2003, 86, 94, 240–42; *Grundlagen*, §31).

20. Dummett, 1981, 4; see 9–12, 240, 334, 557, 576, 677–79.

21. This is a retraction of my 1996 (65–73). My 2003 corrects my mistake. See note 28.

22. On functions, see 1971d, 47, 47 n.†; 1971b, 31, 38–39; on incomplete senses, see 1971c, 54; 1971f, 134; 1968, 538.

23. See also Frege (1979g, 254–55) on the correspondence of sense- and reference-incompleteness.

24. William Marshall notices this (1968a, 300). This by itself does not imply that senses are not objects; indeed, Marshall holds that senses are objects. But it does pull the realm of senses out of the realm of customary references, *pace* Dummett (1981, 680). Thus Marshall can only admit senses as indirect objects. But what content does “indirect object” add to the content of “sense”?—Can indirect objects be incomplete?

25. Dummett omits tones and forces from his list of Frege’s objects. On his own showing, his only other options are that they are functions or nonentities.

26. I thank Richardson, who was then a graduate student in chemistry, in my (1979, ii). The chemistry metaphor is, of course, realist in import.

Dummett speaks of “logical valency” (1981, 32, 62, 43). But Frege wants *metaphysical* valency to unify the thought, resolving—not dissolving—Bradley’s regress, *pace* Dummett (1981, 174–75). Frege is a realist on universals. Frege’s concepts are *ante rem* universals, *pace* Dummett (1981, 257–58). A concept name always refers to literally the same concept regardless of how many objects are truly said to fall under that concept, and even if no objects fall under it. Predicative senses are *ante rem* universals too. They remain literally the same across different thoughts. And assertoric force is a universal, since it remains literally the same across assertions; the judgment stroke always indicates the same force. Insofar as tones concern particular acts of utterance and mental ideas, they are particulars; but *tone* is a universal. Thus while all concepts are universals, not all universals are concepts. We must not confuse the object-concept distinction with the particular-universal distinction. This is not a difficulty. Consider that ideas are particulars and *idea* is a universal.

Even if Frege has another sense of “saturation” in mind, such as solubility (*Grundlagen*, §80), surely it would still be a physical sense, and consistent with

molecular theory. I doubt he has in mind, say, the saturation of a poetry reading with emotion. Marshall shows the limits of several physical senses of Frege's other main metaphor, "incomplete" (1968, 253–58, 262–65). As Dummett notes, metaphors necessarily have limits (1968, 277). Frege agrees (1979e, 137), and adds that any bodily metaphor has limits when applied to immaterial senses (1971h, 134). Thus even the valency metaphor has limits, though the filling of electron-spaces by electrons is remarkably apt. Indeed, Frege's logical atomism is more aptly called his logical chemistry.

Molecules can be *conceptually* sliced in different ways, *pace* Ramsey (Dummett 1981a, 264). For Frege, anything complex can be conceptually sliced in different ways merely by regarding something different as the subject. Water is sliced as H₂O (dihydrogen oxide) or as HOH (hydrogen hydroxide, or hydronium hydroxide); HOH is physically more perspicuous. Frege's example is that it is "more appropriate to represent . . . hydrogen . . . by . . . H [and hydroxil by OH] than . . . to designate hydrogen as de-oxidized hydroxil" (1979b, 37; see 36; 1979d, 194 on chemistry).

Dummett says mathematical functions provide the "really important" metaphor of Frege's functions (1968, 296). I agree, but only as to mapping values. Frege expressly generalizes the mathematical notion into his notion, but circularity is not a problem at the level of explication. The problem is illuminating the problem of metaphysical unity. I see no illumination of the metaphysical problem, regardless of the clarity of the formal notation for function names, and regardless of whether mathematical functions are themselves instances of the functions to be explicated. What literal incompleteness of mathematical functions would be the basis of the metaphor illuminating the literal incompleteness of Frege's functions in general? What would be metaphorical about it?

27. Frege 1971i, 64, emphasis mine; quoted twice in my 2003, 65, 68. "Ein Wahrheitswerth kann nicht ein Theil eines Gedankens sein, sowenig wie etwa die Sonne, weil er kein Sinn ist, sondern ein Gegenstand" (Frege 1892, 35). Literally, "A truth-value cannot be a part of a thought, as little as for example the Sun, because it is no sense, but rather an object." Or according to *Cassell's*, "it is no sense, but on the contrary, an object."

28. Thus I was wrong to suggest that senses should be assimilated to the first model, with "the sense of 'A'" referring to an object which represents the sense (my 1996, 68–73; 1982, 13; 1981, 36–37). For Frege openly says it refers to the sense (my 1996, 69; 1981, 36). My earlier idea was feasible by "converting" senses to objects, but the representation would not be mutual, i.e., one-one, due to sensial underdetermination (*pace* my 1996, 118; 1982, 13). My 2003 corrects my mistake. Of course, we could arbitrarily *posit* a representative object that corresponds one-one to the sense, but Frege clearly does no such thing, since he says we refer to the sense.

29. In the schema, "meaning" means reference. Truth-values, of course, are objects.

30. Nor does Frege write "concept" for the incomplete sense, for it is not a concept but a sense.

If senses are objects, why does Frege fail to say so in his schema of his view?

Frege says, “The . . . schema should make my view clear.” If senses are objects, how could the schema possibly make his view clear?

Does Frege, perhaps, fear confusing Husserl by explaining the additional complexity that the senses too are objects, though none is the object the proposition in question is about? I would think that if senses were objects, *not* saying so would be confusing. Indeed, the schema should eliminate any such confusions. Otherwise it would not be making Frege’s view clear. This is exactly the sort of thing Frege should be labeling in the schema and explaining in the letter. It would have been easy to do. I explained it very easily myself just now. He could even have said “indirect object.”

Does Frege feel it is too obvious to mention that senses are objects? Other things in the schema are far more obvious, such as that proper names refer to objects. And how obvious is it that senses are objects?

Does Frege feel it is too unimportant to mention that senses are objects? The omission of such a basic categorial relationship would be monumental.

31. Frege says persons are objects (1971b, 31), making them customary references. Perhaps he thinks of persons as perceptible and bodily. Perhaps persons are a class of bodies, namely bodies having minds. But then persons ought to be emergent entities on the double take, since they or their minds are also the homes of ideas. Their emergent properties would include public acting and speaking.

32. Explications seem best glossed as emergent entities on the doubletake as well, since they aim at meetings of private minds on public references via public senses.

Senses might be emergent entities too. Frege describes only one constituent, the mode of presentation each contains. But they have a linguistic property of being expressible by names that this nonlinguistic, merely cognitive constituent lacks. I would like to say that the other logical constituent of senses is their completeness or incompleteness in conformity with the context principle. Their emergent property would be that of explaining how informative identity statements are possible.

I do not know what Frege would have thought of my glosses of emergence. He should be familiar with the concept of emergence. Numbers (classes of classes) have emergent properties, such as being odd or even, which their constituent classes lack. Whether these properties are ultimately purely logical does not matter to this point. There is also his statement that definitions can result in something new (1974, 100–101); and definition implies logical complexity, i.e., logical constituents. But Frege never says that senses, forces, tones, persons, names, or explications are emergents; and if they are, that would be a most important generalization to communicate. Nor does he define them, though he often seems to state their constituents and the relationships among their constituents. He may say so little about them because he knows they are mere logical emergents, or because he does *not* know and finds them mysterious, or because he takes them to be primitive categories, or because he takes them to be what they do. Of course, there are other possibilities; he may simply be laconic. Opinions may also differ on how much these entities’ being emergents would remove the mystery about them. At least they would be not *deus ex machina sui generis*, but relational states of affairs logically composed of reasonably familiar constituents.

33. Dummett rejects Saul Kripke's thesis that every sense is descriptive, since Frege says that without some use of contextual cues, we could not convey where or when anything is (Dummett 1981a, 84; see 85–128; Frege 1979f, 213). But this confuses the sense with the conveyance of the sense—with “the conditions of utterance” (Frege 1968a, 517). Indexicality concerns only the linguistic garb of senses. In ordinary language, explications are the norm. They can convey references only via senses. Their indexicality is fourfold: *I* must guess what *you* mean *here* and *now*. Surely they are often ostensive. Kluge says Frege does not discuss this (Kluge 1971, xxviii). But Frege mentions “the pointing of fingers” (1968a, 517). Frege divorces senses themselves from indexicality of any kind. They are timeless and placeless. Surely they are logically tied many-one to equally timeless and placeless concepts under or within which the references (if any) fall. Consider Frege's example, “the pupil of Plato . . .” (1971i, 58 n.*). Dummett is right that in practice, not every “sense can be conveyed by means of a definite description” (1981a, 85). But the operant sense of “can” is logically can. Also, a sense can always be conveyed by a description used referentially in Donnellan's sense.

34. The sense-reference distinction is not so basic to logic that Frege could not have completed his formal logicism in 1884, *pace* Dummett (1995, 8). Even a “mere label” name-theorist can define numbers as classes of classes. And Frege distinguishes contents from ways of presenting contents in *Begriffsschrift*. Thus he had a “glimmering” of the distinction after all, *pace* Dummett (1995, 7). Indeed, his earlier ways of presenting contents would seem to be what later mutate into senses. I agree with Dummett that *Grundlagen* would be clearer if it expressly used the distinction, and that a philosophy of logic, as opposed to a formal logicist system, ought to make some such distinction.

35. This is directly so for Frege, since his truth-values *are* referential objects.

36. Frege's implicit nod to intuitionism is his requirement of existence-proofs. He says that to prove a concept's consistency, we must produce an object falling under the concept (*Grundlagen*, §§74, 95 n.1); and we cannot existentially quantify over an inconsistent concept to yield a truth.

Frege even says “*F*'s exist” *means* that there is at least one *F* (*Begriffsschrift*, §12 n.15), i.e., that there are not zero *F*'s (*Grundlagen*, §53). But this is no general theory of intuitionistic meaning. It is merely his analysis of the single word “exist,” and his informal analysis at that. Frege formally defines the existential quantifier in terms of the universal quantifier and negation, and he makes it clear that we understand “all *F*'s” even if we do not know every *F*. Both quantifiers express universal senses and refer to universal concepts.

37. One need not be a phenomenologist to agree that: “By saying that we do not or cannot understand [impossible] suppositions, we can only mean that we find what they suppose unthinkable, inconceivable. For, of course, we do not literally mean that we do not or that we cannot understand them; what we mean is that we cannot *think* what we do understand them to suppose. . . . Inability to understand, in the sense in which philosophers appeal to it when determining the absolute impossibility of something, is the *unthinkability* of what, in a sense, is *understood* to be described by the proposition, not the literal nonunderstanding of that proposition” (Butchvarov 1970, 81). Frege agrees, analyzing thinkability in terms

of references, and understanding in terms of senses (1971i, 58). But intuitionists are constrained to literal nonunderstanding of nonintuitionistic mathematics, on pain of its being as literally meaningful as intuitionistic mathematics.

38. Neither Frege nor Russell says so, but probably they both believed that every truth of logic is either an axiom or a provable theorem. Certainly the Frege-Russell camp was stunned by Gödel's incompleteness proof. But neither Frege nor Russell makes proof/disproof a requirement of meaningfulness. Frege says it may not always be in our power to apply a concept, and Russell expressly rejects intuitionism.

39. The pre-sense-reference Frege says, "That a concept contains a contradiction is not always obvious without investigation; but to investigate it we must first possess it and, in logic, treat it just like any other" (1974, 87). Frege adds, "The concept 'fraction smaller than 1 and such that no fraction smaller than one exceeds it in magnitude' is quite unexceptionable: in order, indeed, to prove that there exists no such fraction, we must make use of just this concept, despite its containing a contradiction" (1974, 87 n.1). On the sense-reference split, the point applies not only to the concept, but to the sense as well. For Frege we must first possess the sense, and in order to prove that there exists no reference, we must make use of just this sense (1971i, 58). Ironically, it is the question whether any mathematical objects exist that leads Dummett to hold that we must first possess the reference in order to prove that there exists any sense to our inquiry.

If I were to defend intuitionism, I would look to Parmenides' view that nothing can be said or thought about nothing. This implies that we must first possess the reference. But that reduces Parmenides to taking the backward road. Also, like the verification principle, Parmenides' view violates itself. It says something about nothing. And the sense-reference distinction is Frege's reply to Parmenides.

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